

# Neutrino Mass Ordering and the Neutrino-nian Knot



\* There's not a lot of Alexander the Great clipart.  
Hercules will be his stand-in.



Keith Matera

**FNAL Undergraduate Lecture Series — July 26, 2016**

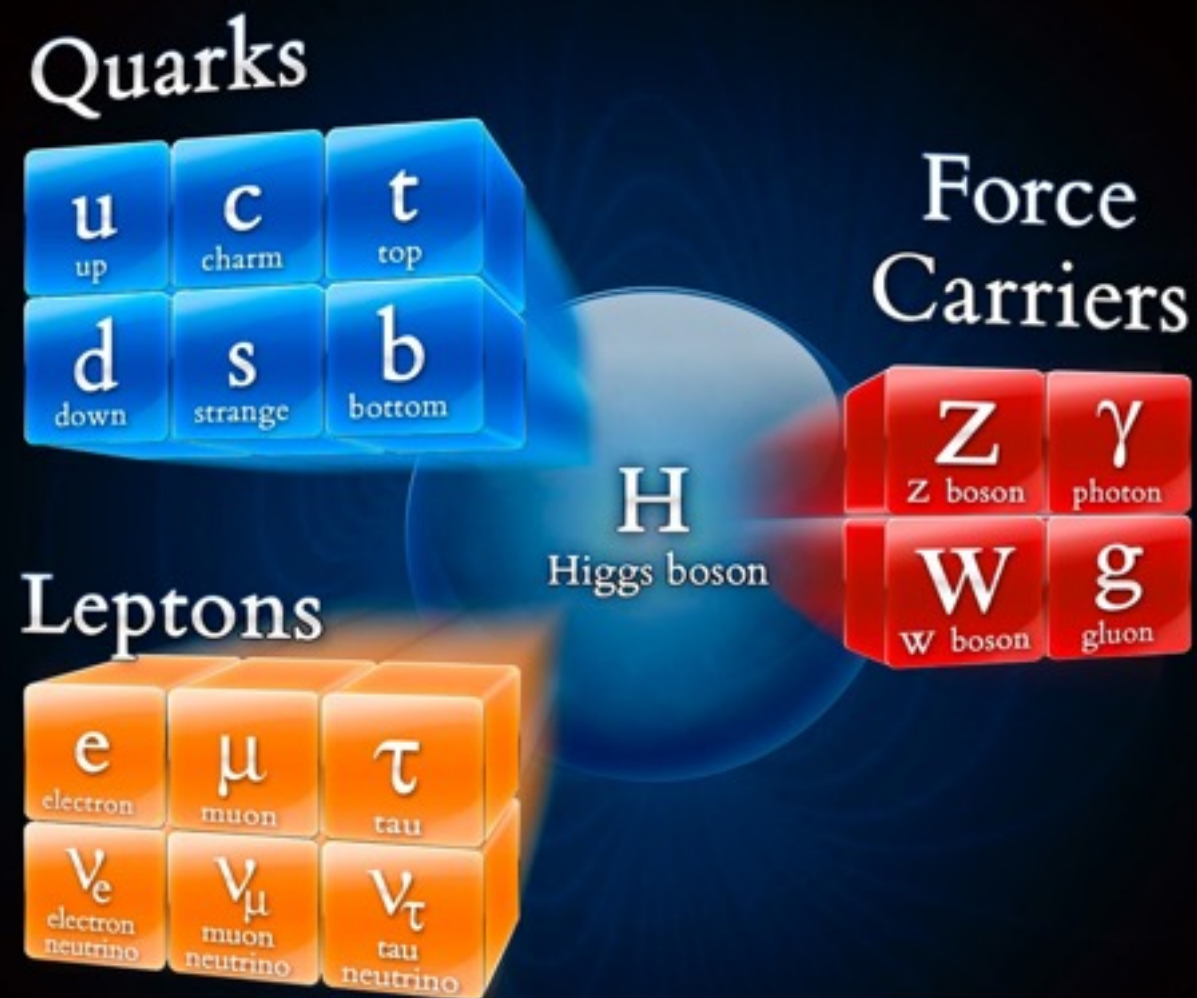
# PART I

## Neutrinos!



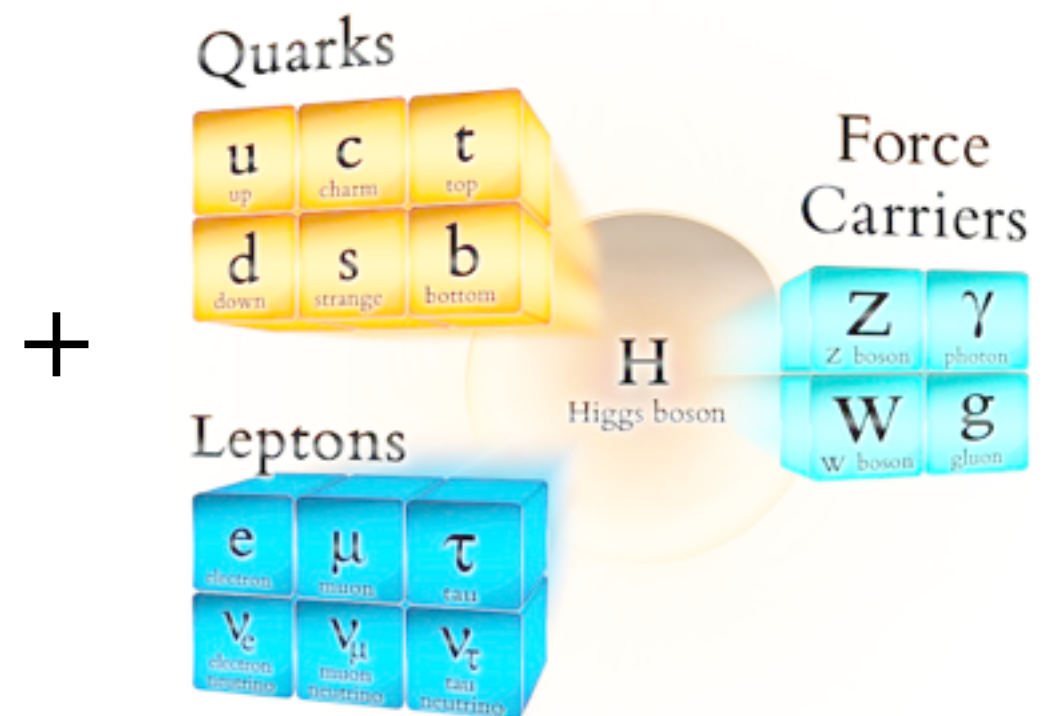
# The Standard Model

The world map with which we set off on our quest.



All of the (known)  
elementary particles.

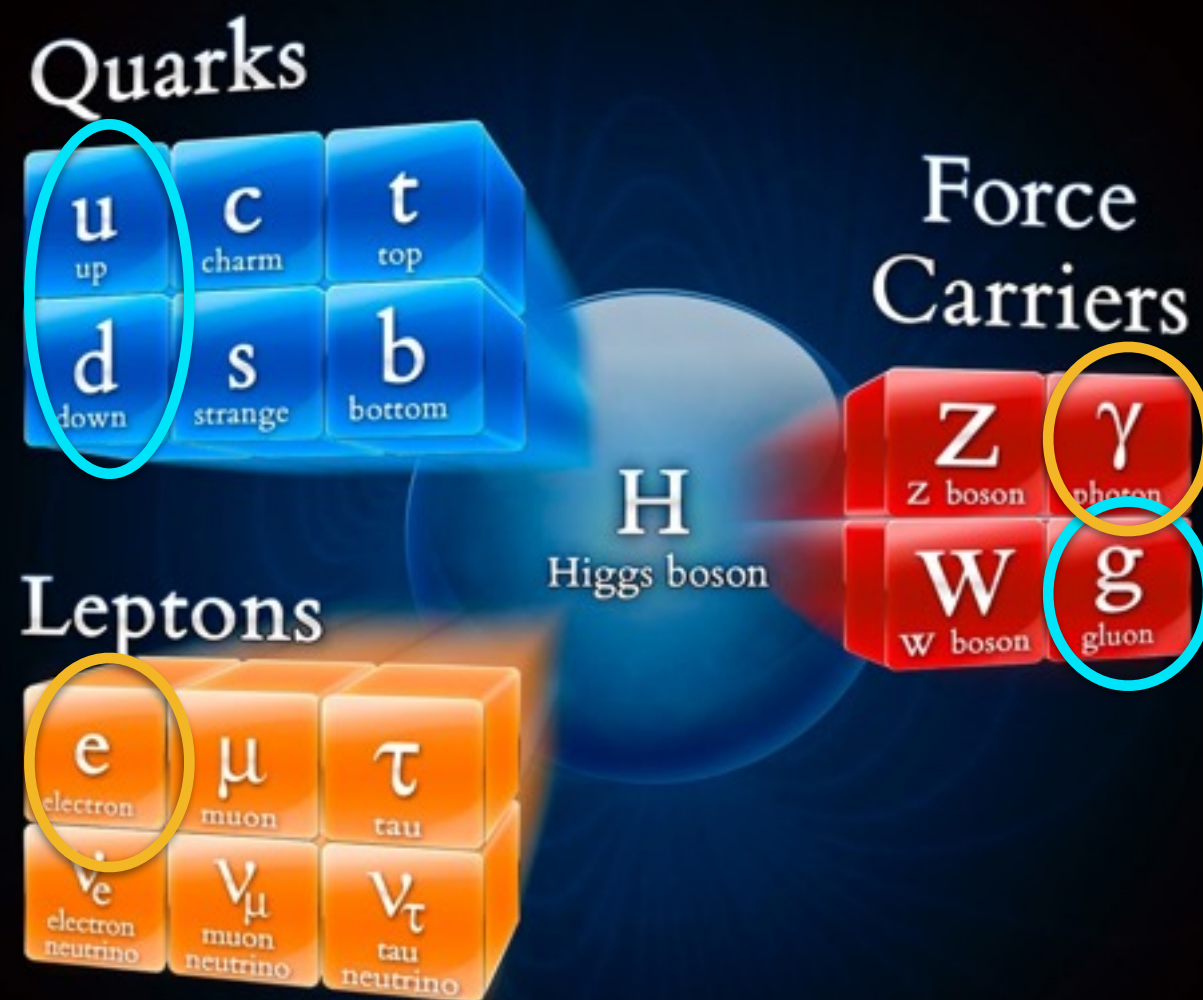
Anti-matter



\*Disclaimer: Some of these particles (e.g., the photon) are their own anti-particles.

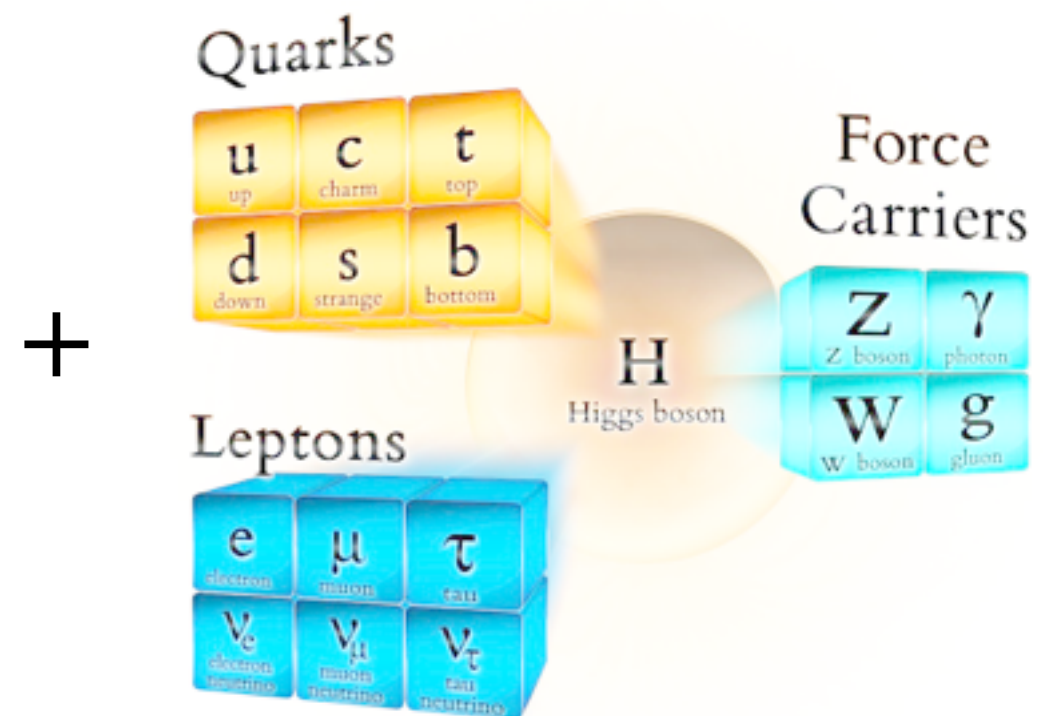


# Everyday matter consists of only a small piece of the standard model



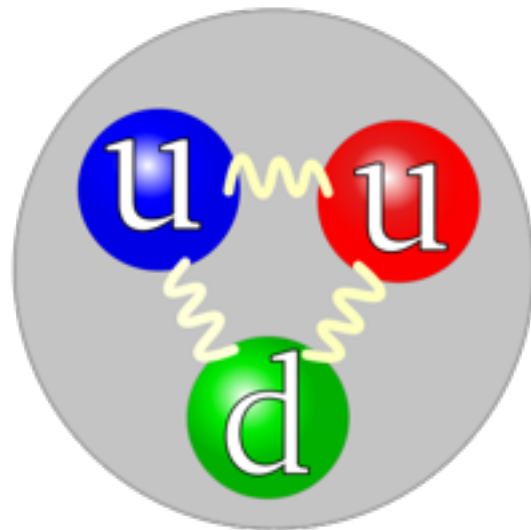
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Anti-matter

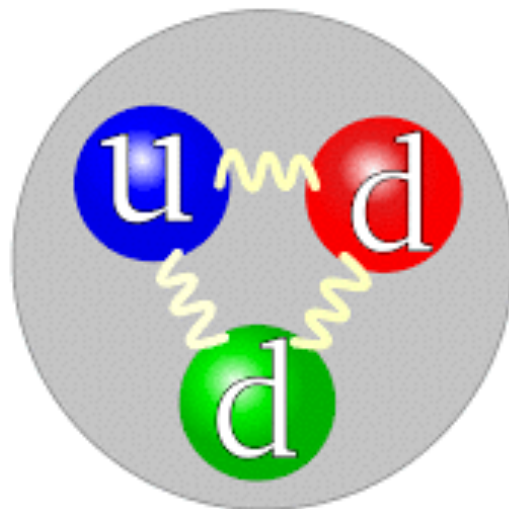


\*Disclaimer: Some of these particles (e.g., the photon) are their own anti-particles.

# Up and down quarks give us protons and neutrons

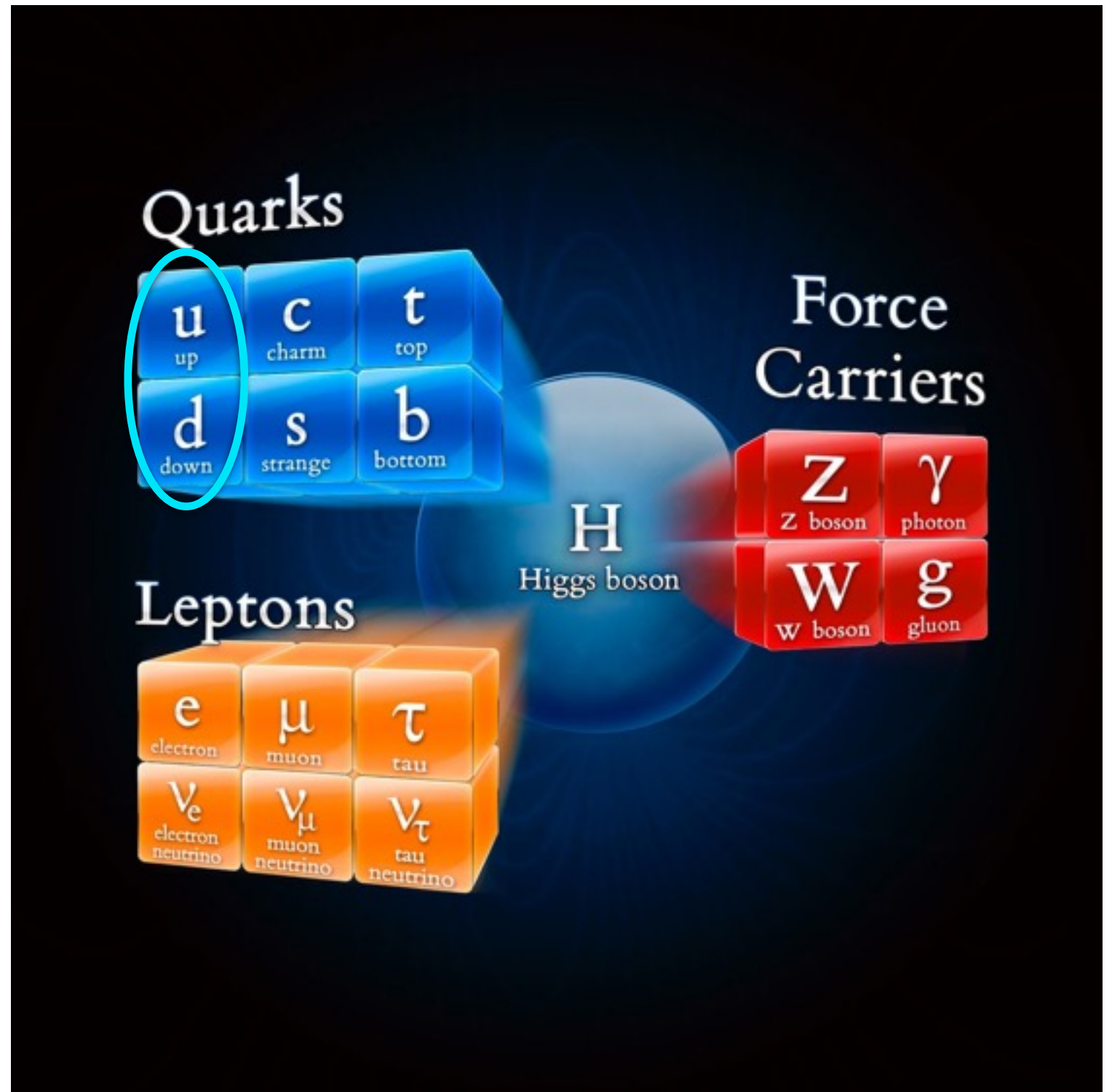


Proton

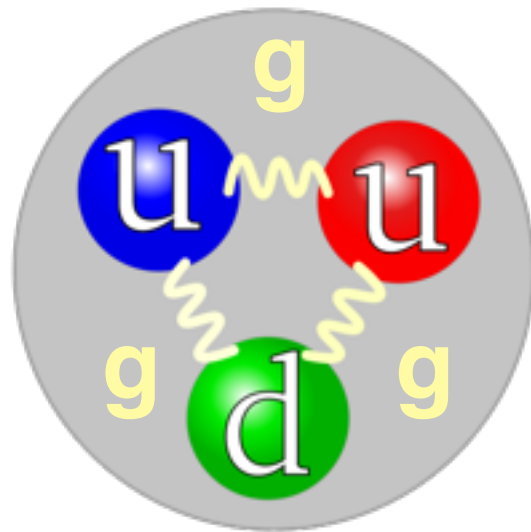


Neutron

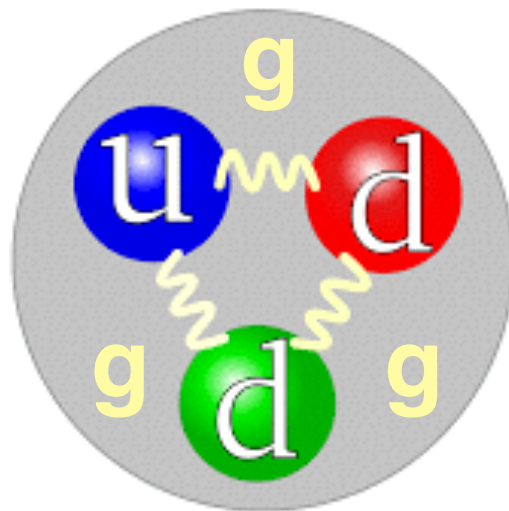
Image Source: [Wikipedia](#)



# They are held together by the gluon (strong force carrier)

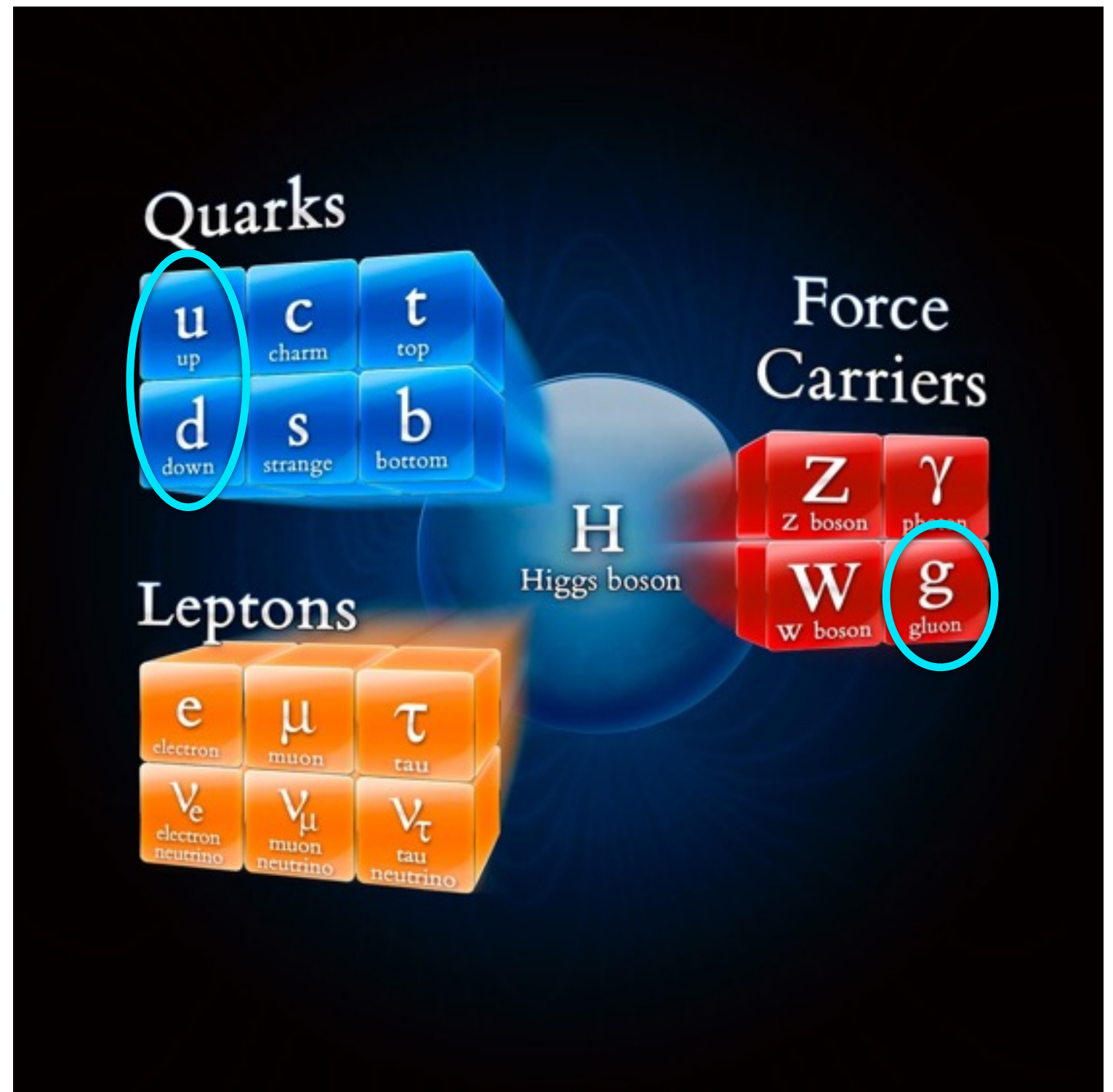


Proton



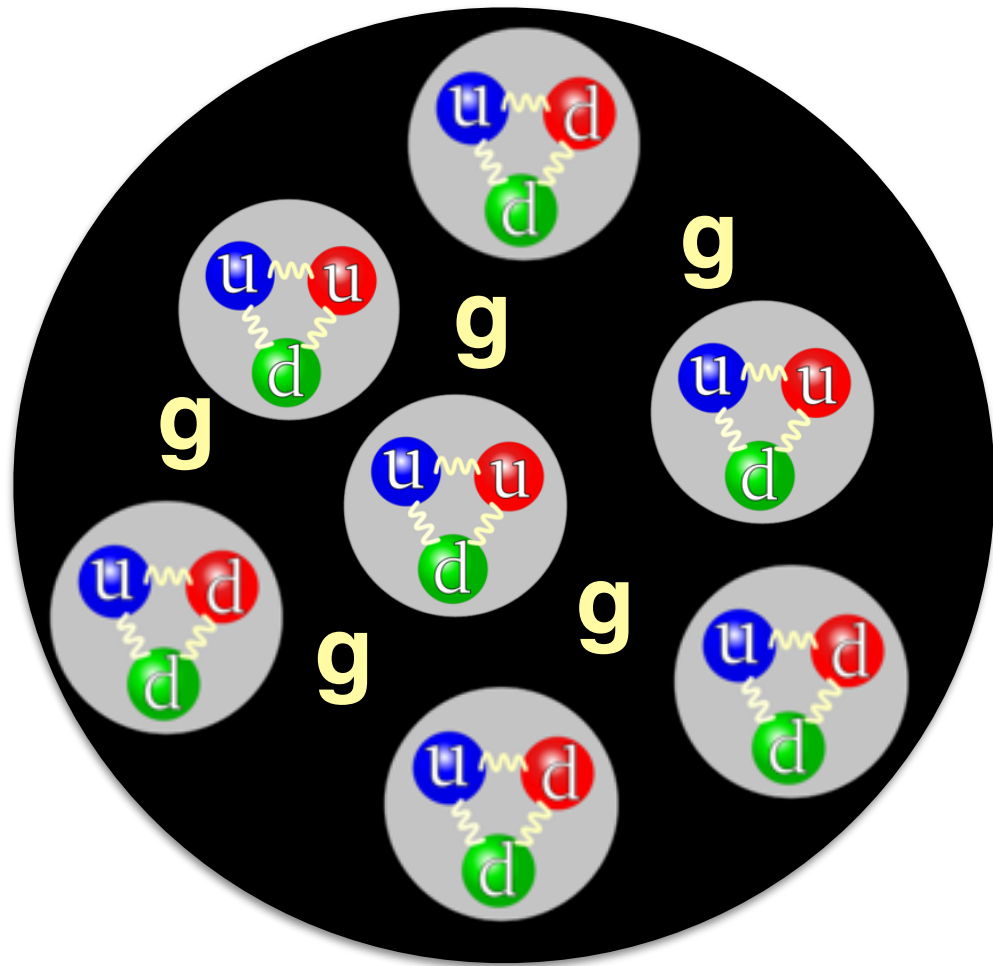
Neutron

Image Source: [Wikipedia](#)

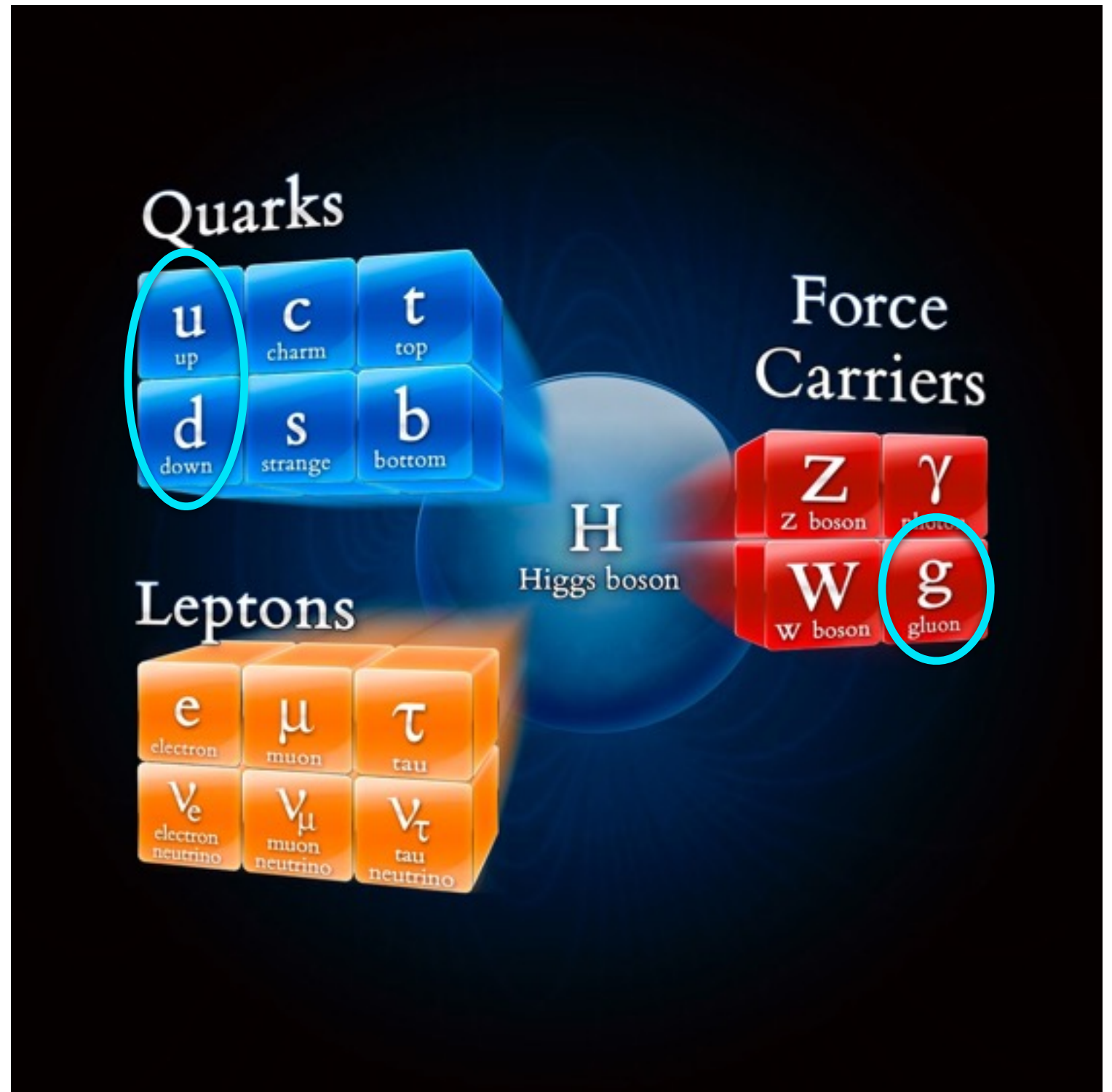




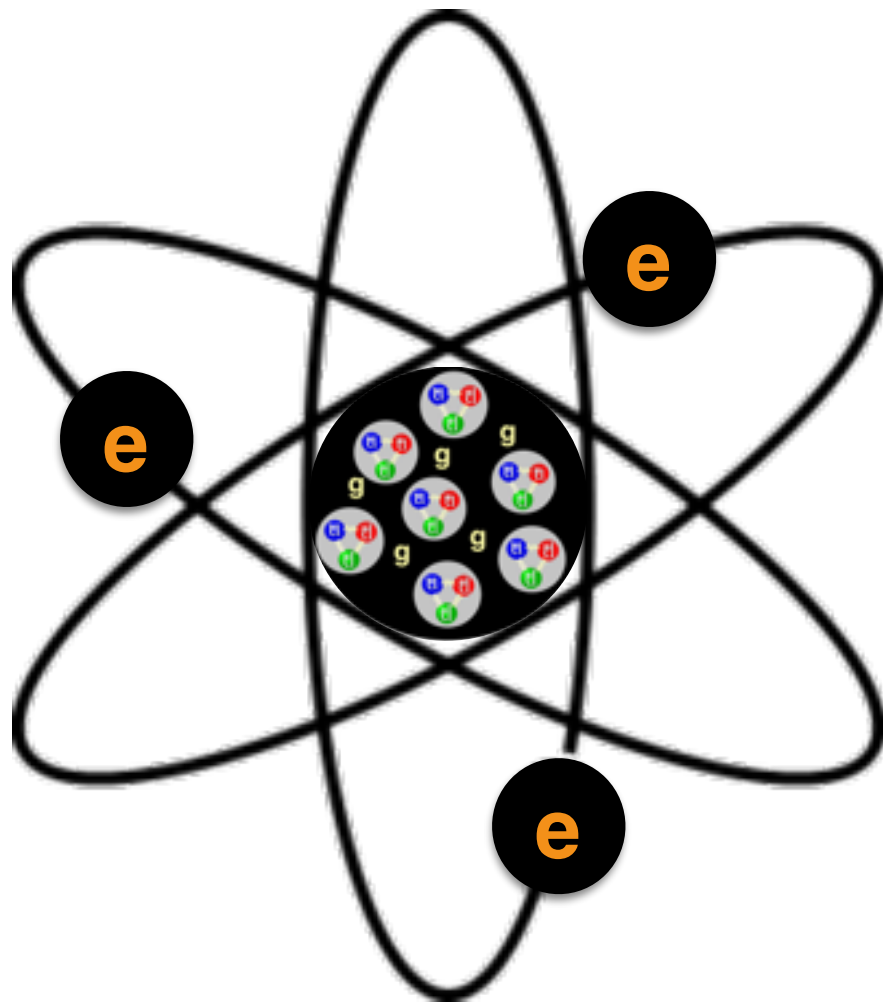
# Residual strong force keeps protons and neutrons bound together



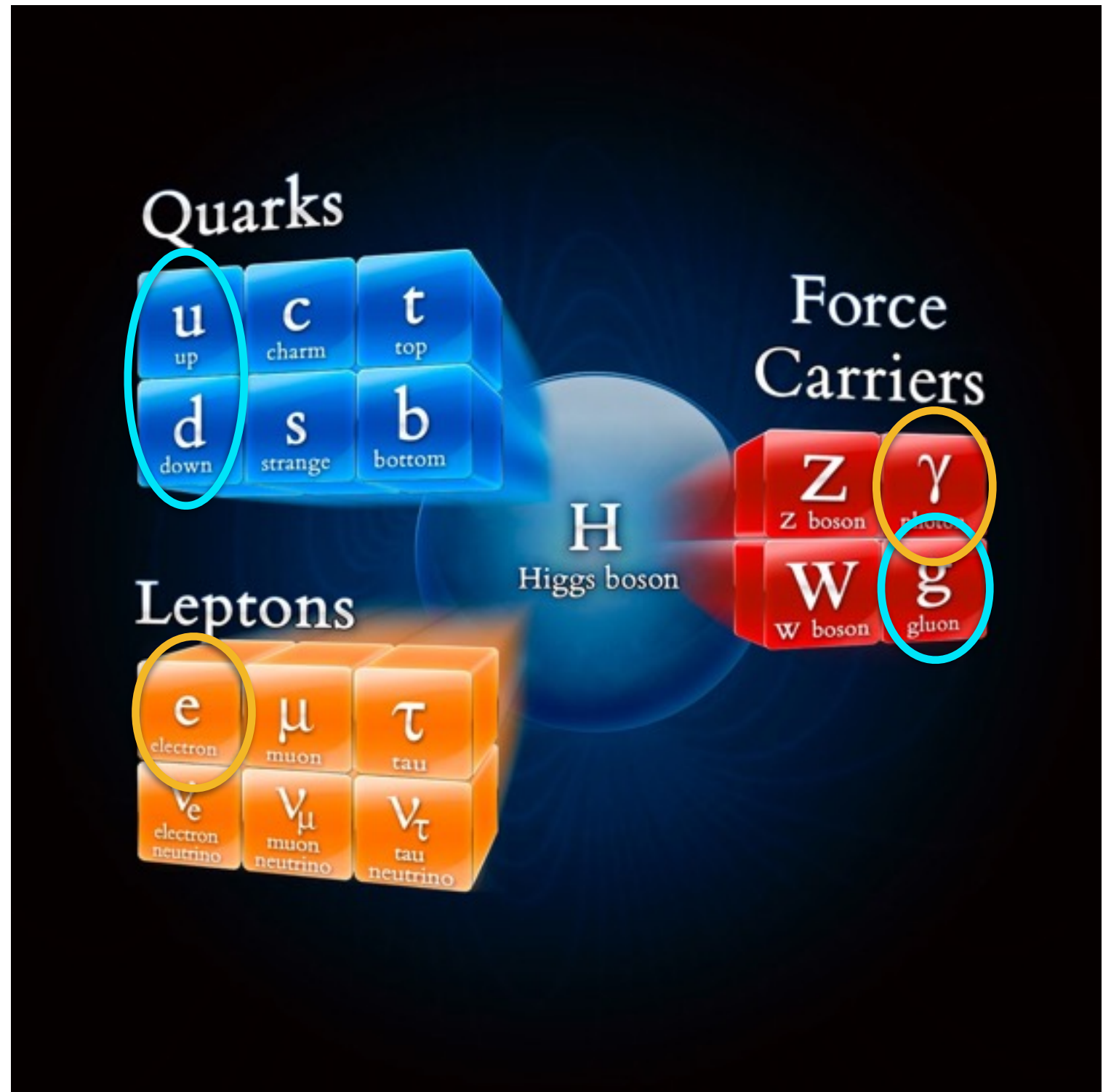
Nucleus



# And electrons are bound to nuclei by photons (electromagnetic force carrier)

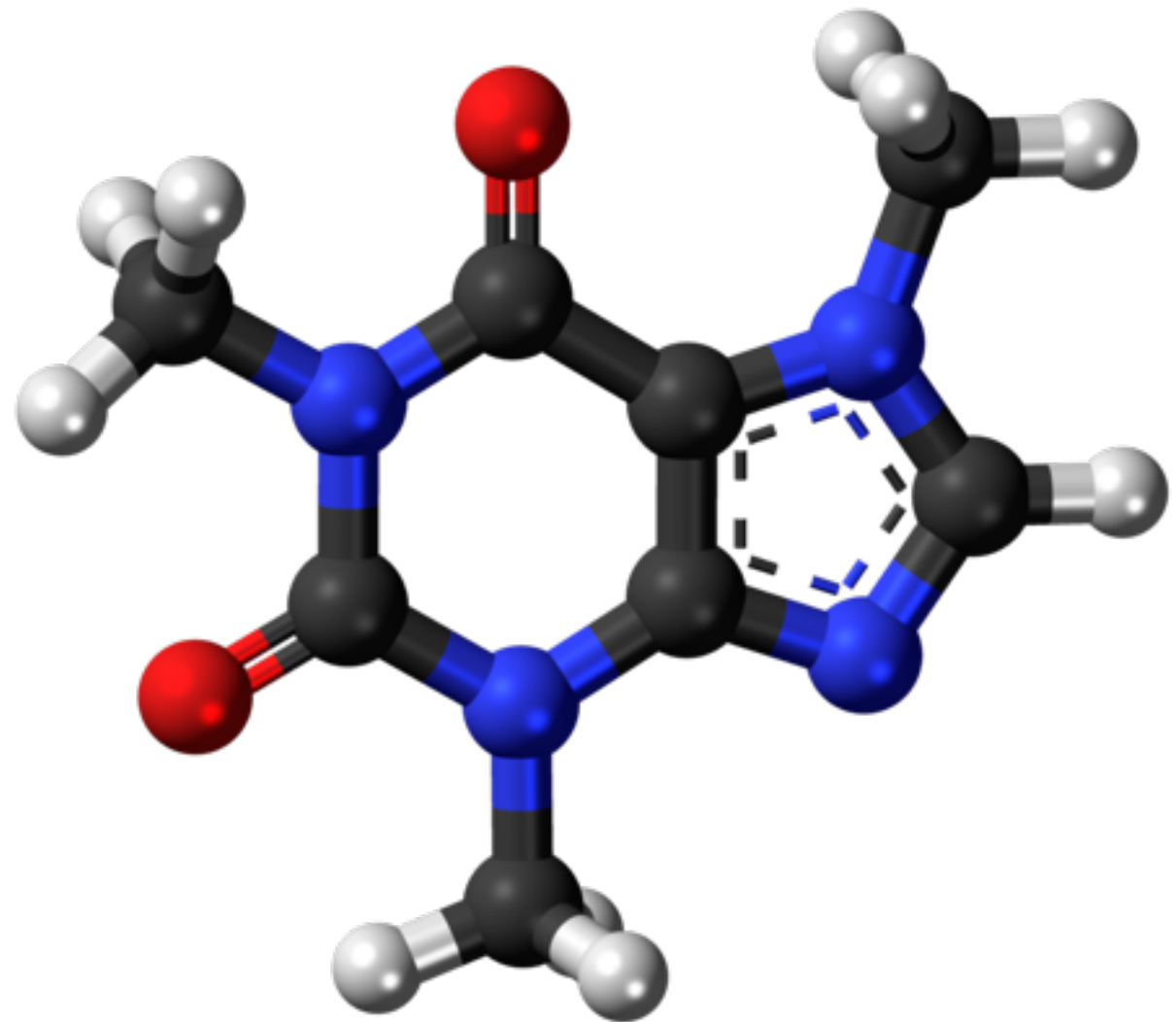
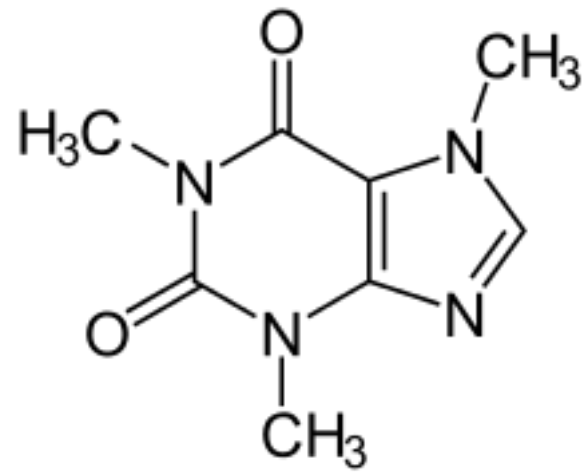


Atom



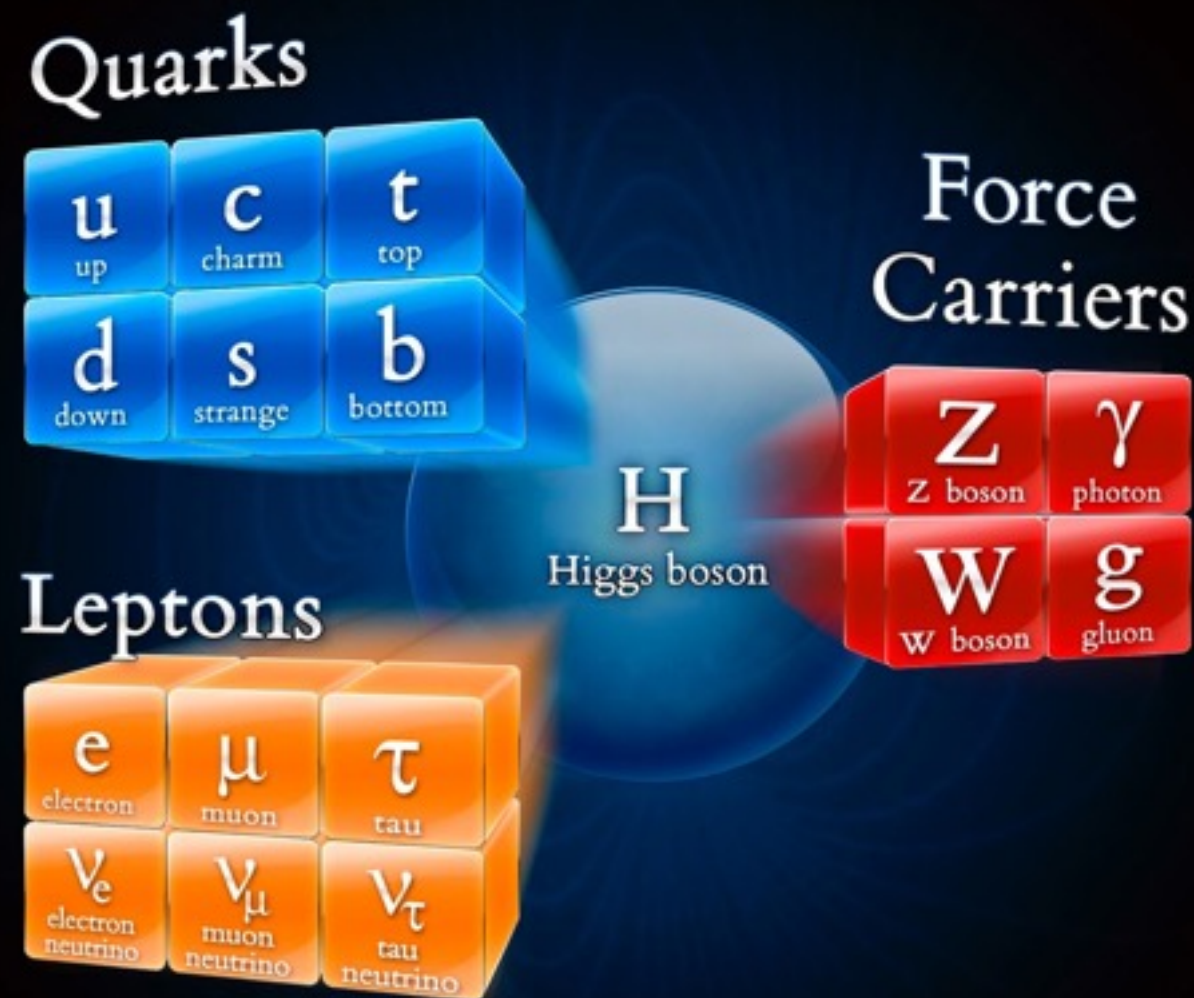


These atoms interact electromagnetically to give us the chemicals that make up our world

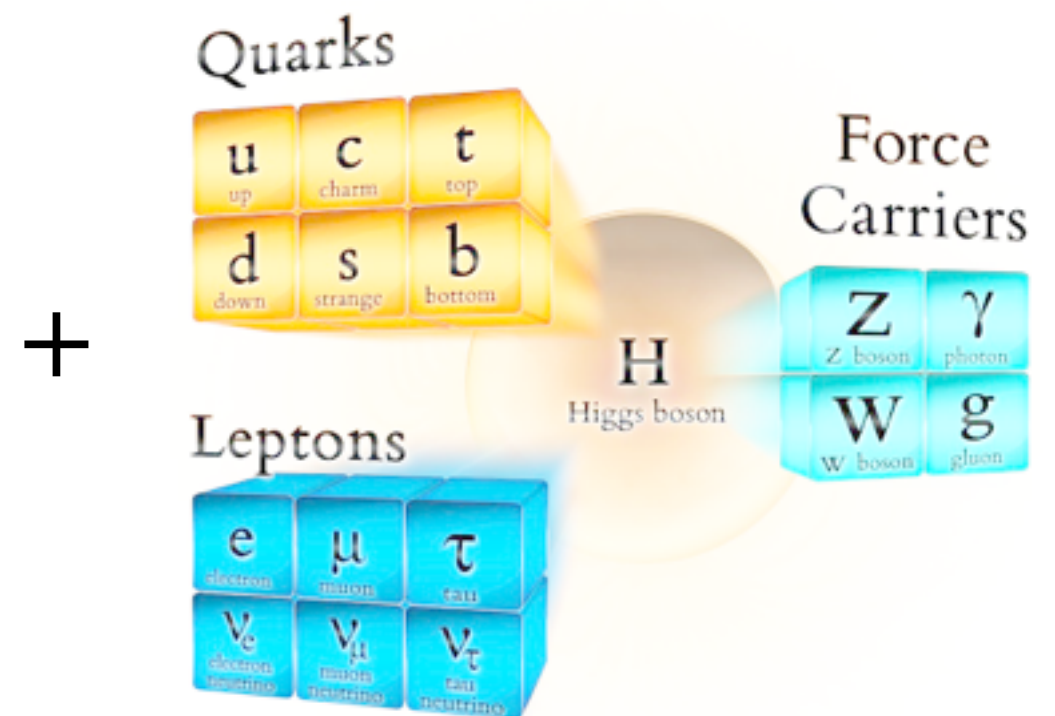


Images: [Wikipedia, "Caffeine"](#)

# But what about the rest of the elementary particles?



Anti-matter



\*Disclaimer: Some of these particles (e.g., the photon) are their own anti-particles.



# At the beginning of the universe, these particles existed in abundance!

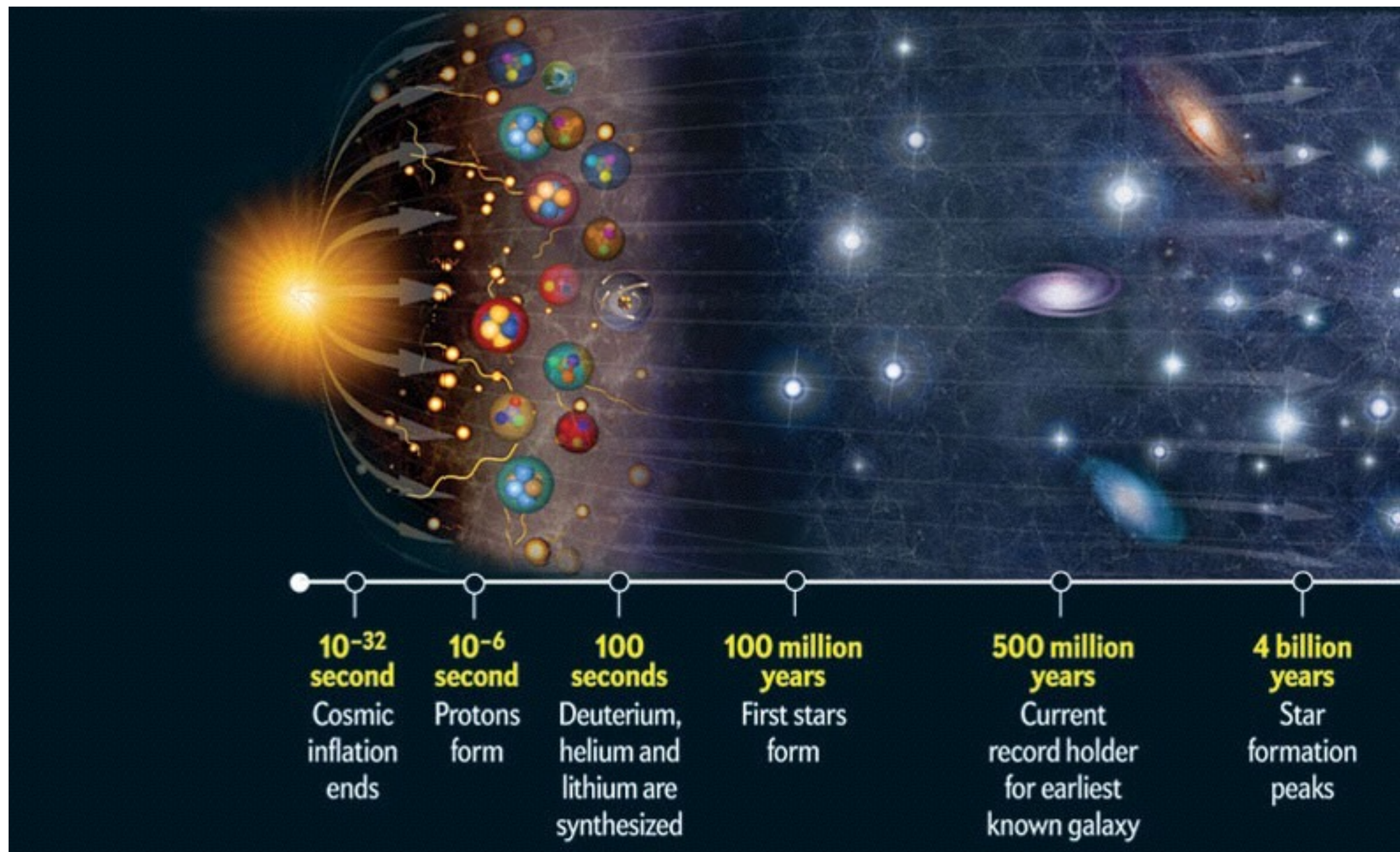


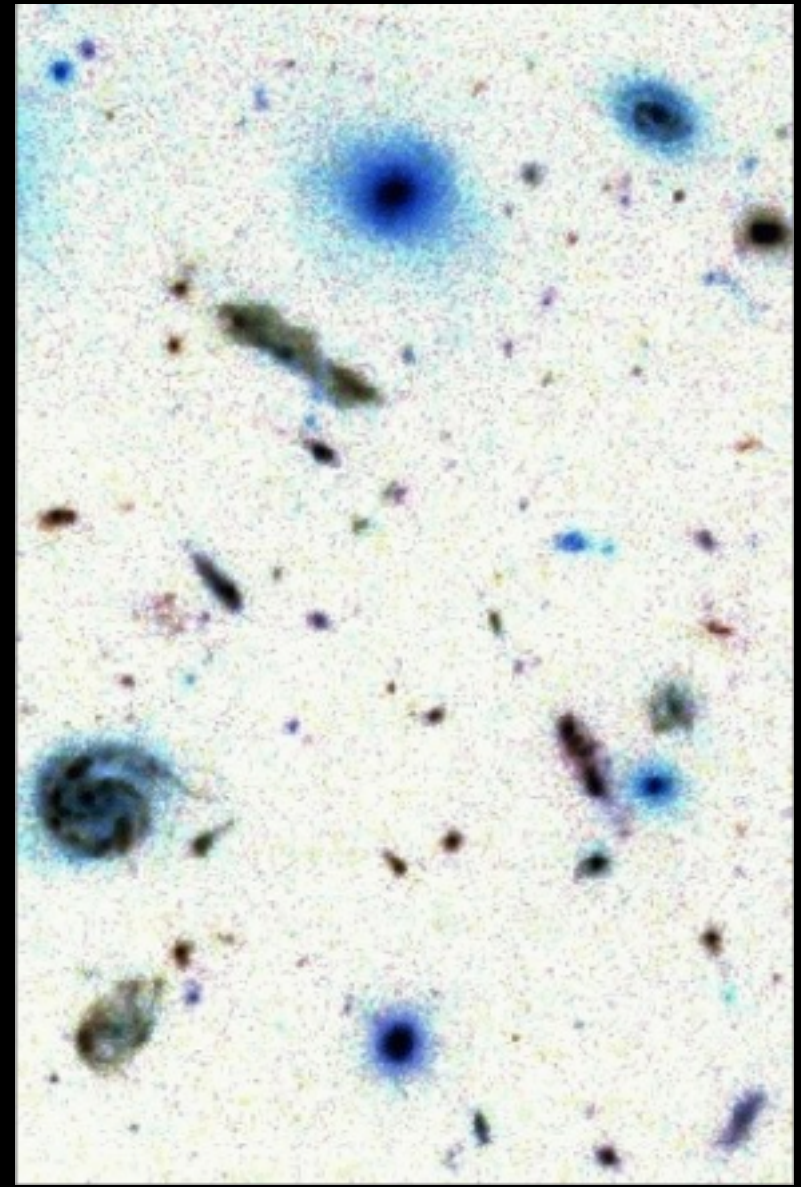
Image Source: [Astroblogs: "Waarin dijt het heelal uit?"](#)



# Matter and anti-matter almost completely annihilated



10,000,000,001



10,000,000,000

# Leaving behind a universe dominated by matter



And as the universe cooled down, heavier,  
more exotic particles were unstable

...and **they decayed away**...

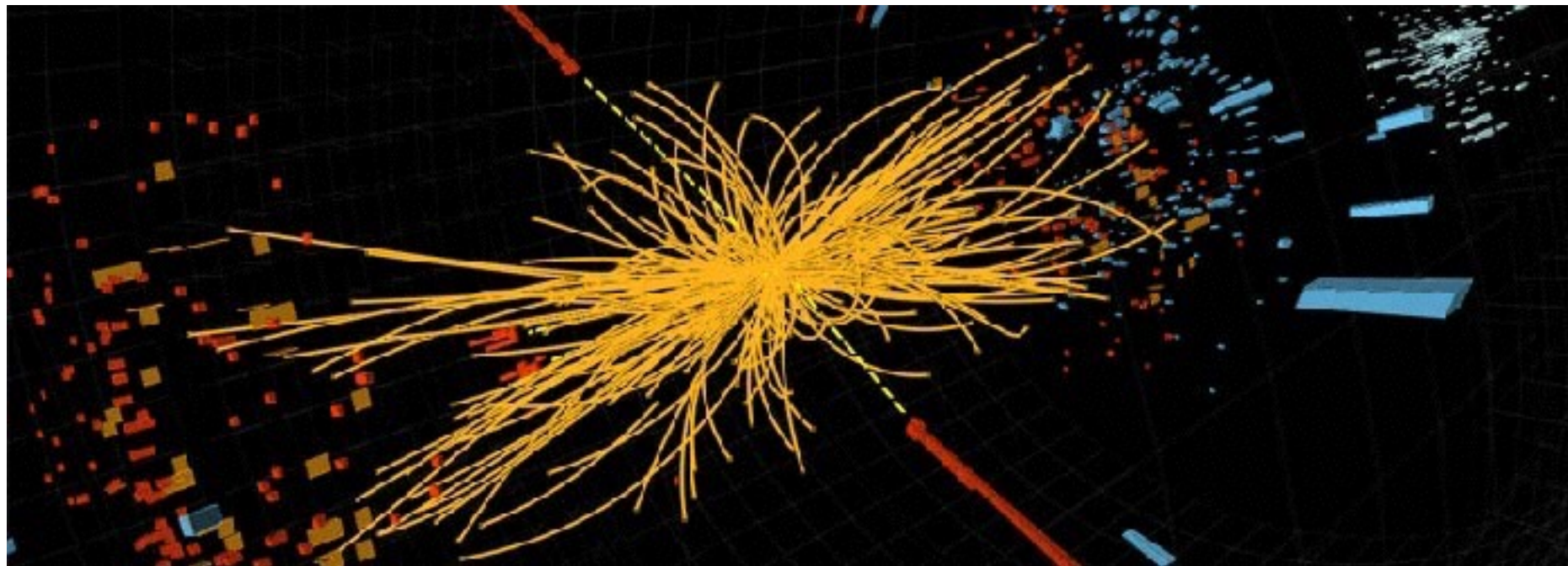


Image Source: [CERN, Simulated CMS Event Display](#)

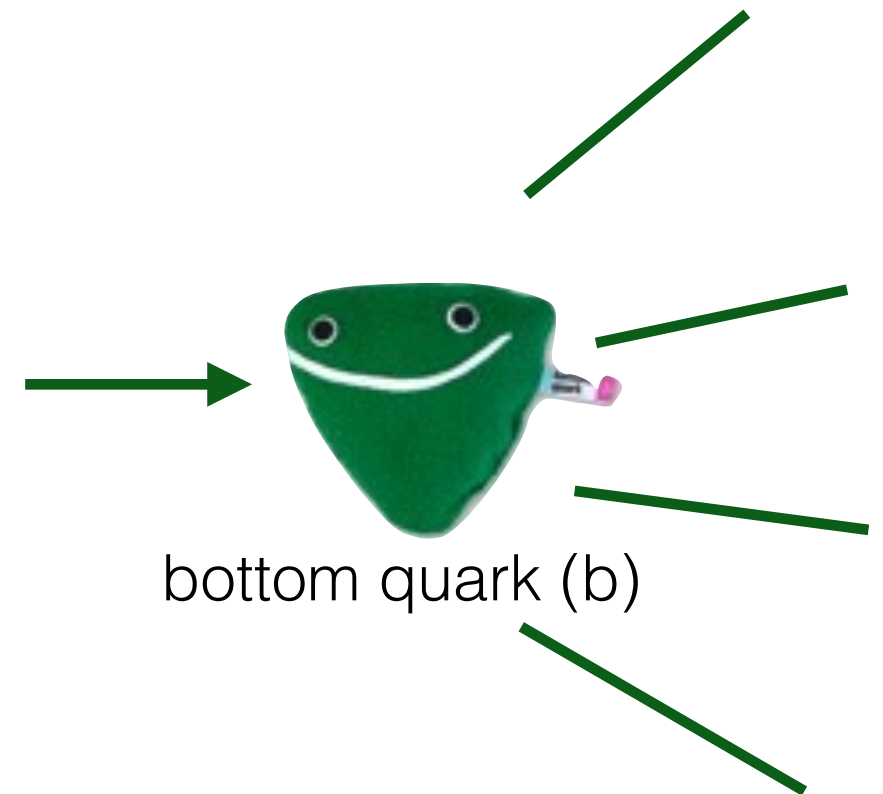
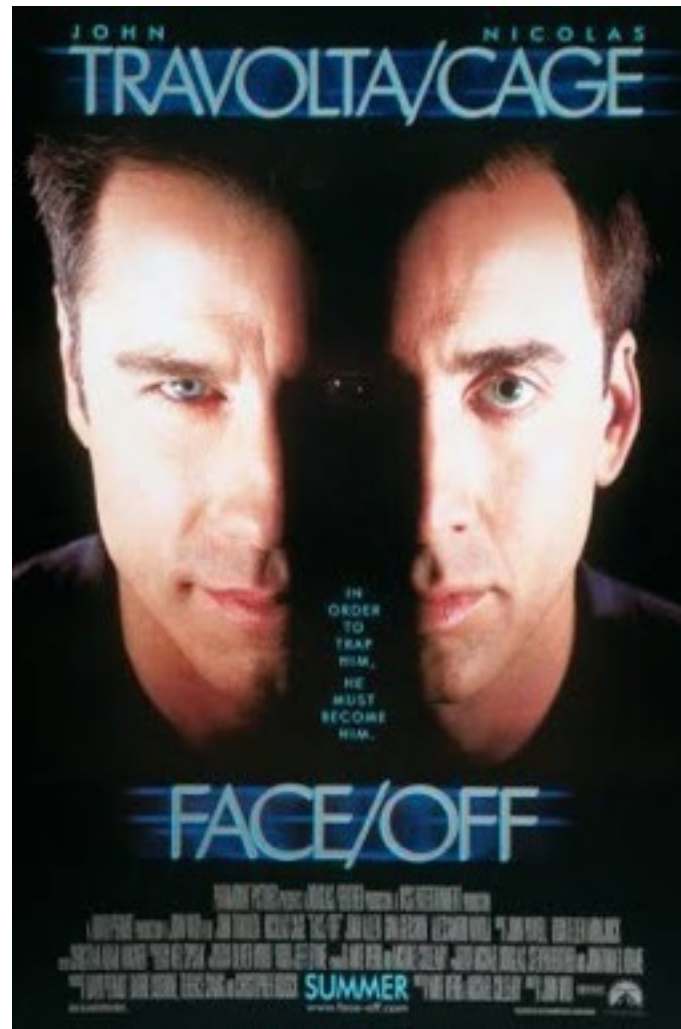
Disclaimer: This is a simulated Higgs production event in proton-proton collisions at CMS. Some of the final-states of particles represented by this image are probably muons. Should be considered for illustrative purposes only.

...until all that was left were the familiar, lighter particles!



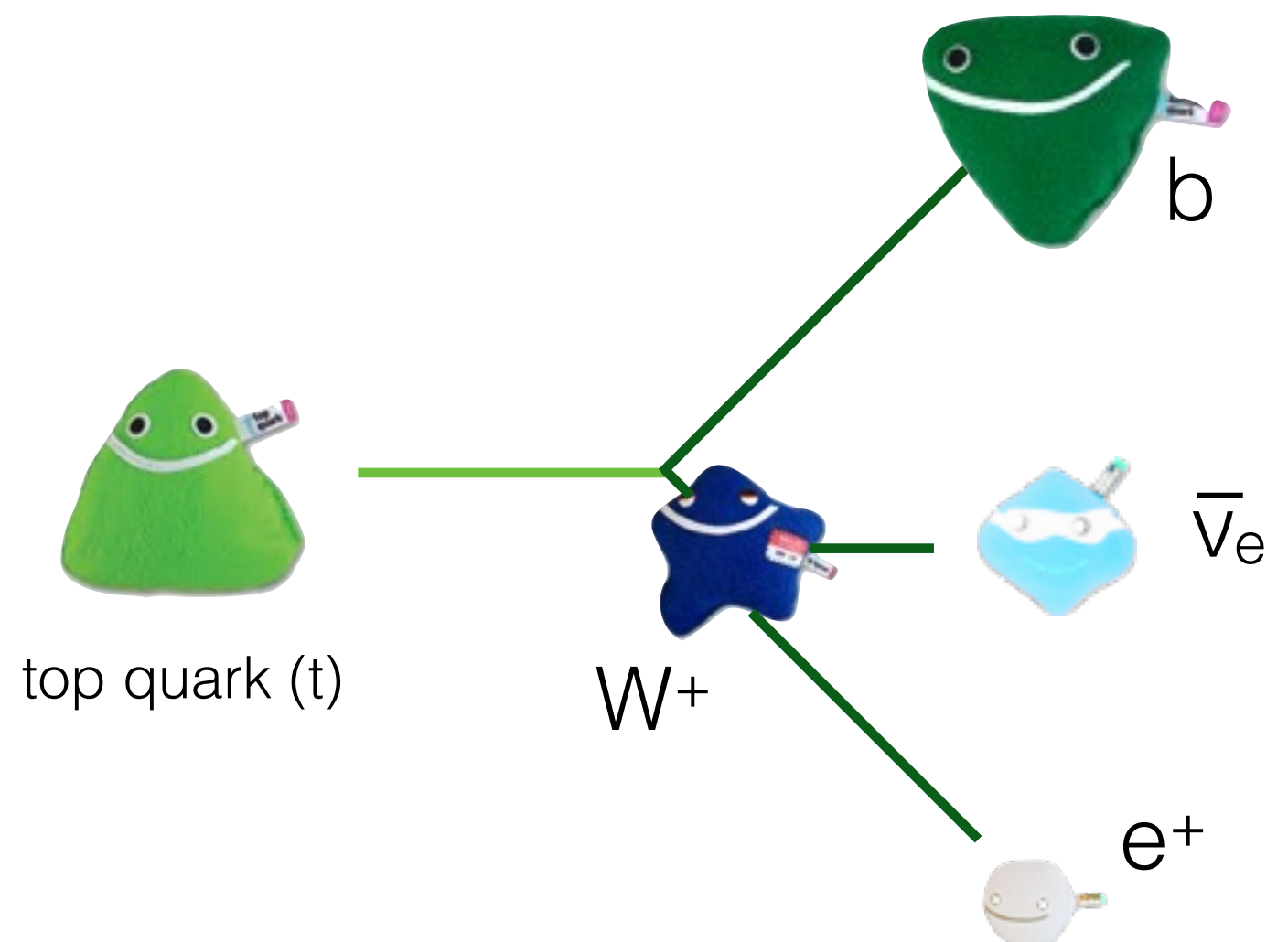
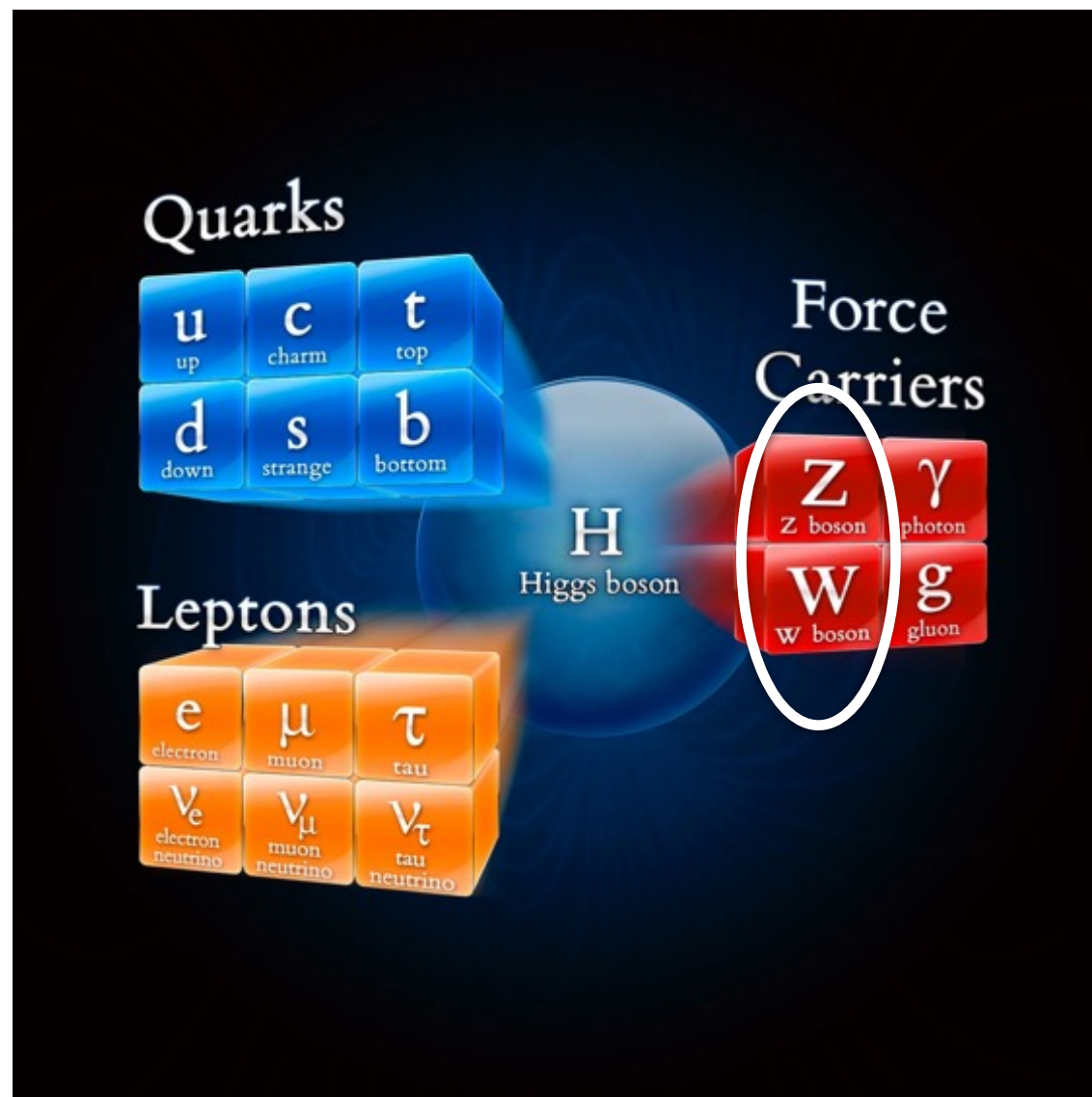
# Elementary particles aren't composed of anything smaller

They **can't simply 'break apart'** when they decay. They must ***change identity*** and shed excess energy (as particles).



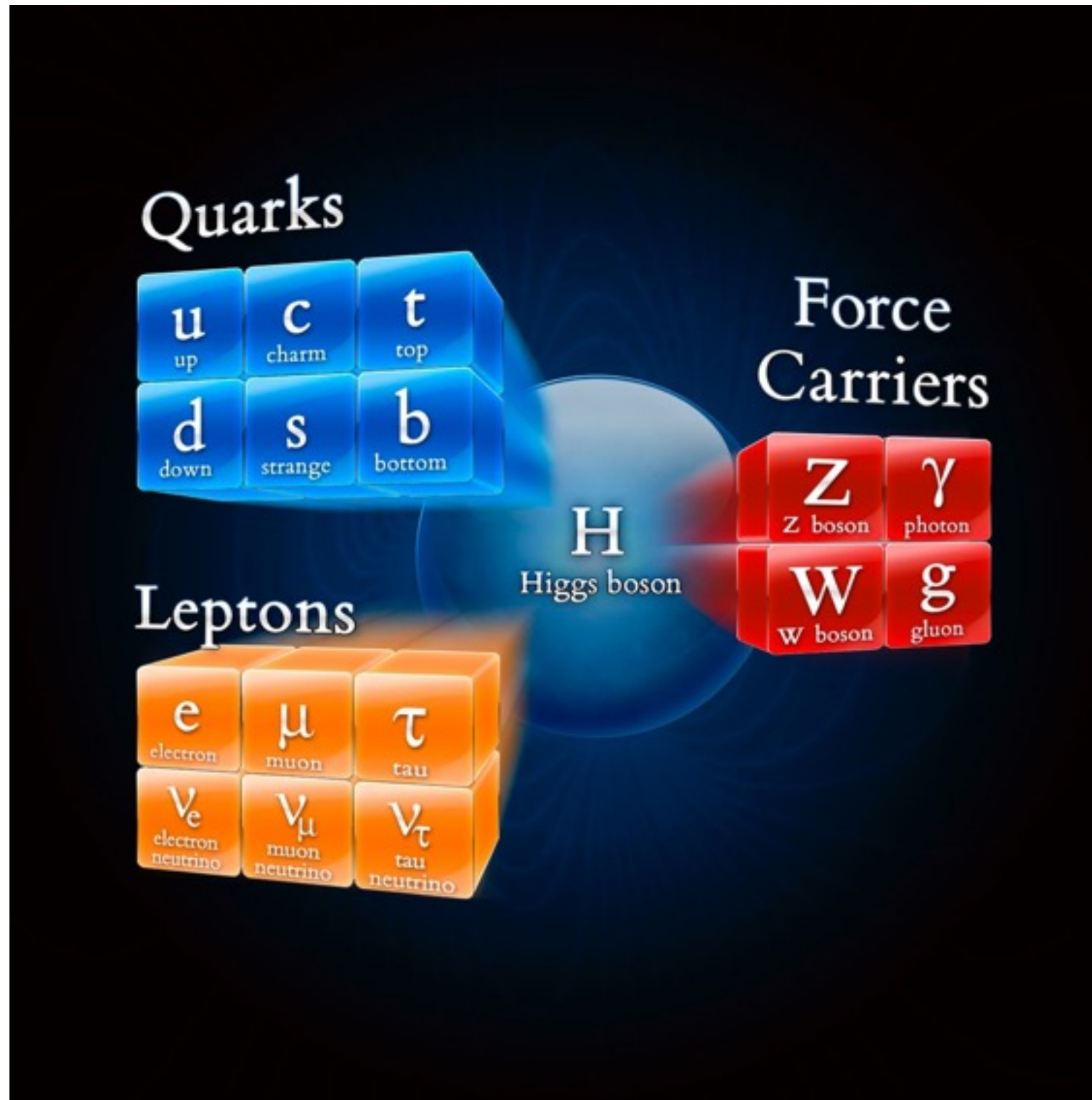
**\*Disclaimer:** Except for the top quark, quarks do not exist outside of bound states. In general, quark flavor-changing interactions would take place inside of a meson or baryon.

# This kind of identity-swapping is possible through the **weak force**



And **neutrinos** are the weak force's attendant!

# So going back to these...



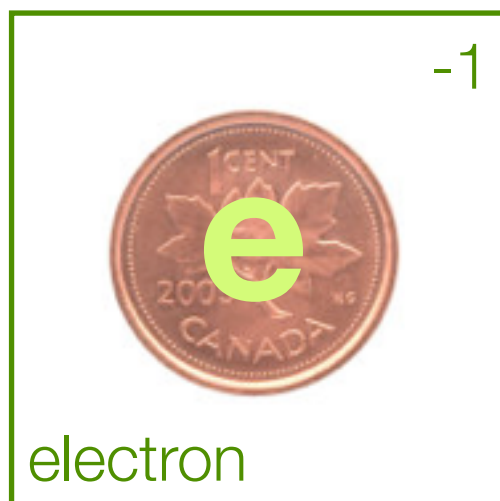


# quarks



All of **matter**  
is made up  
of 'coin'-  
like  
particles

# leptons



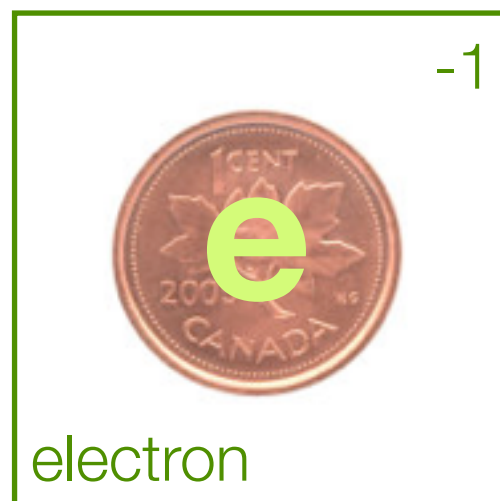
Each  
particle can  
be '**flipped**'  
by **weak**  
**interactions**

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# quarks

$+2/3$



up

$+2/3$



charm

$+2/3$



top

$-1/3$



down

$-1/3$



strange

$-1/3$



bottom

All of **matter**  
is made up  
of 'coin'-  
like  
particles


# leptons

0



electron neutrino

-1



electron

-1



muon

-1



tau

Each  
particle can  
be '**flipped**'  
by **weak**  
**interactions**



# quarks



All of **matter**  
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particles

# leptons



Each  
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# quarks

$+2/3$



up

$+2/3$



charm

$+2/3$



top

$-1/3$



down

$-1/3$



strange

$-1/3$



bottom

# leptons

0



electron neutrino

0




muon neutrino

0



tau neutrino

-1



electron

-1



muon

-1

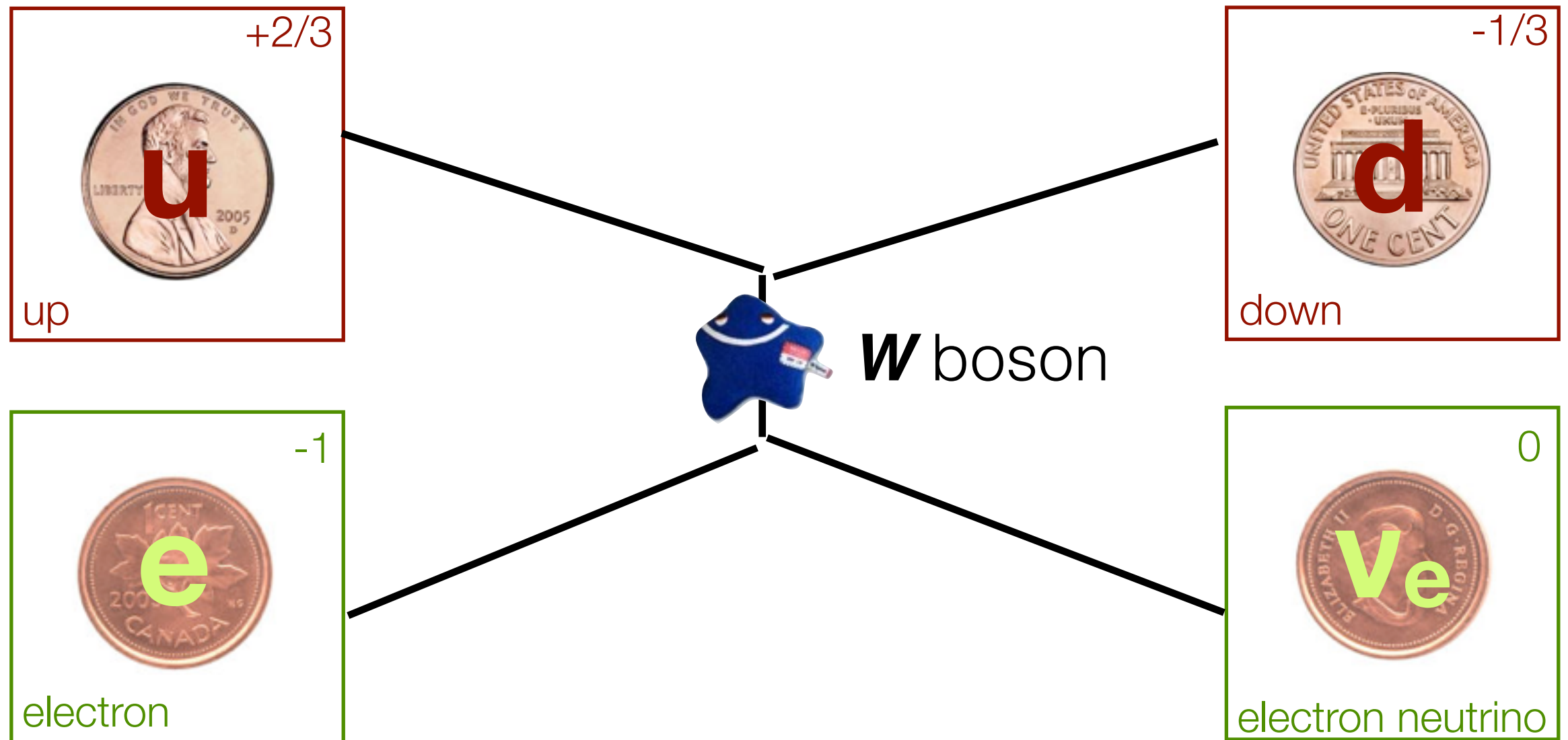


tau

All of **matter**  
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Each  
particle can  
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**interactions**

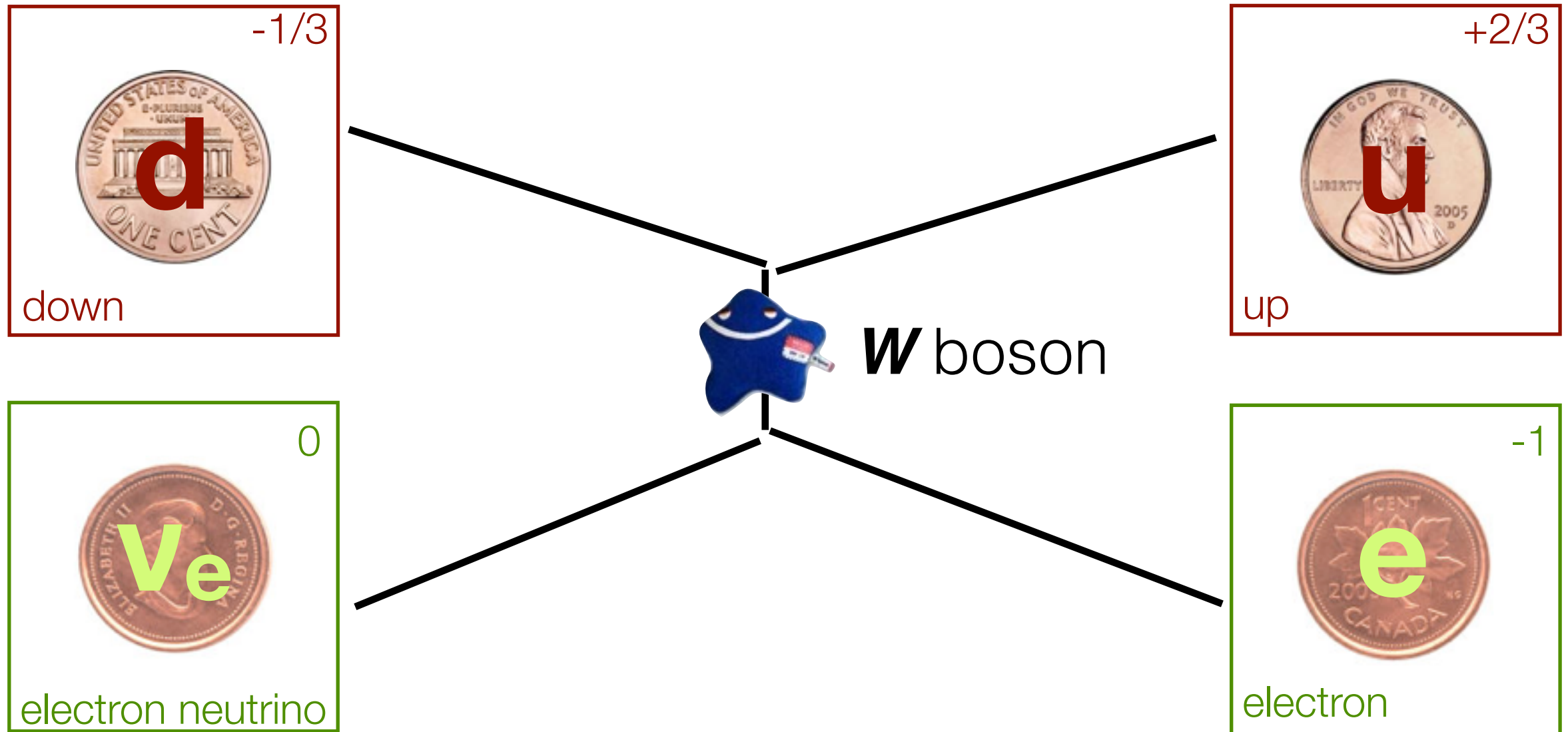
# But these interactions cannot happen alone



An up quark can be flipped into a down quark — but, *e.g.*, an electron must be flipped into a neutrino as well.

**Neutrinos are very frequently involved!**

In fact, we can use neutrinos to force these processes to go in reverse

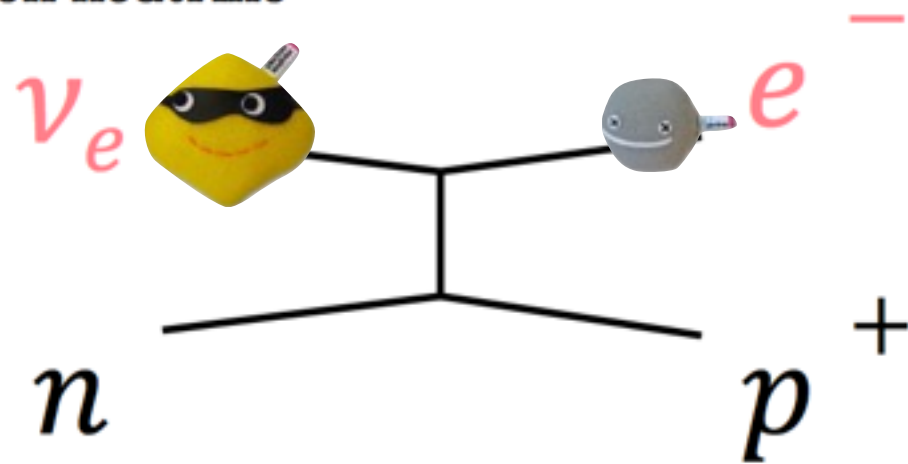


**This is the basis of neutrino detectors!**

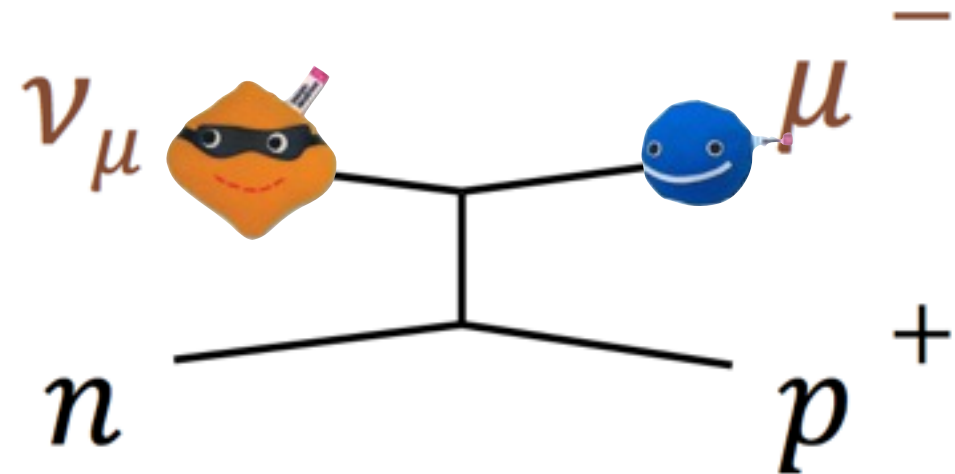


In such interactions, neutrino flavor states will change into the particle they're named after

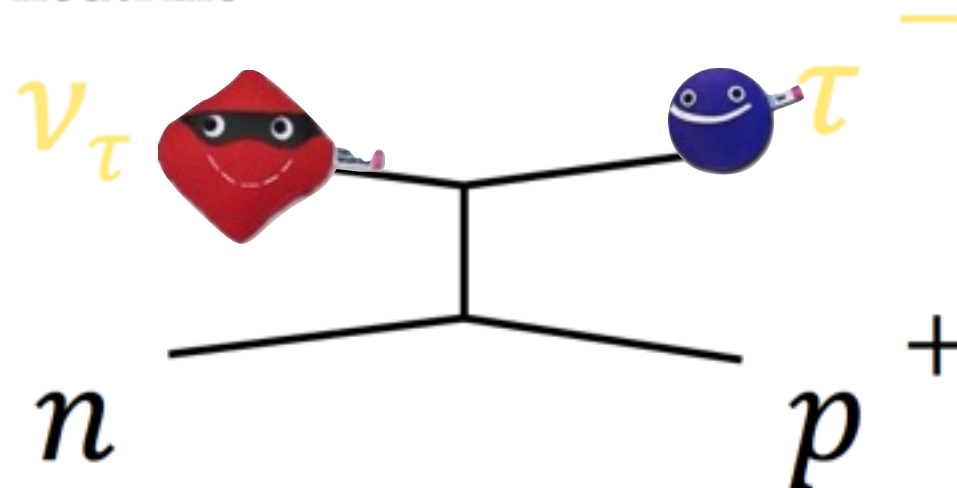
electron-neutrino



muon-neutrino



tau-neutrino



(Diagrams from Anne Schukraft's talk on June 16)

# Neutrinos can change identity without involving other particles

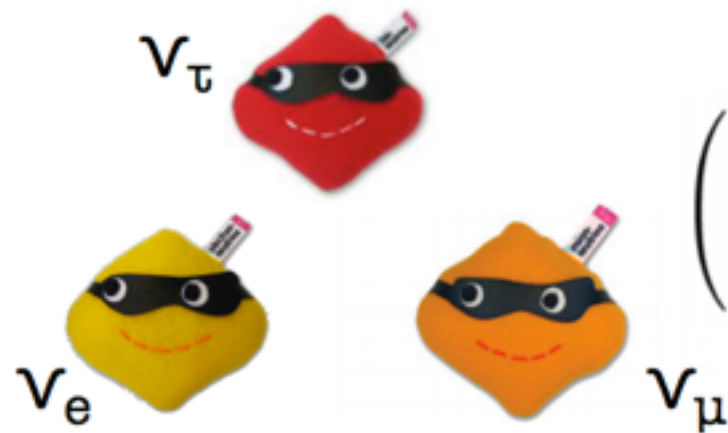
As neutrinos travel, they ***oscillate*** through three identities: electron neutrino, muon neutrino, and tau neutrino.



This means that an electron neutrino created in the sun might be detected here on earth as a muon neutrino!

# This is because neutrino flavor states are composed of different mass states

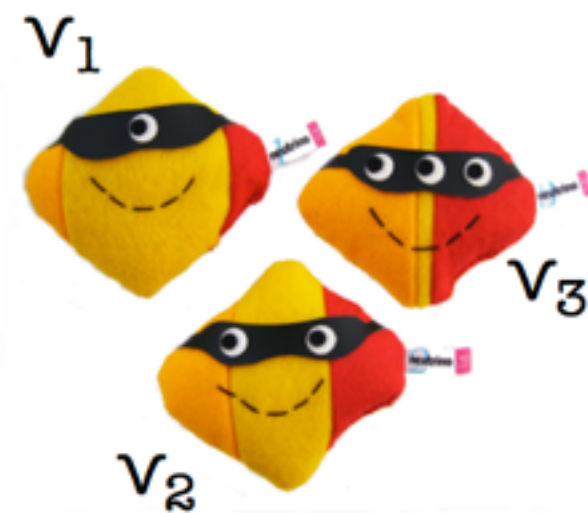
Flavor eigenstates



When neutrinos interact, we see the flavor eigenstate

$\neq$

Mass eigenstates



But they travel (wave function) as a mass eigenstate

(Recall that  $p = mv$ . The same  $p$  applied to three different  $m$  will give three different  $v$ !)

(Graphics from Anne Schukraft's talk on June 16)

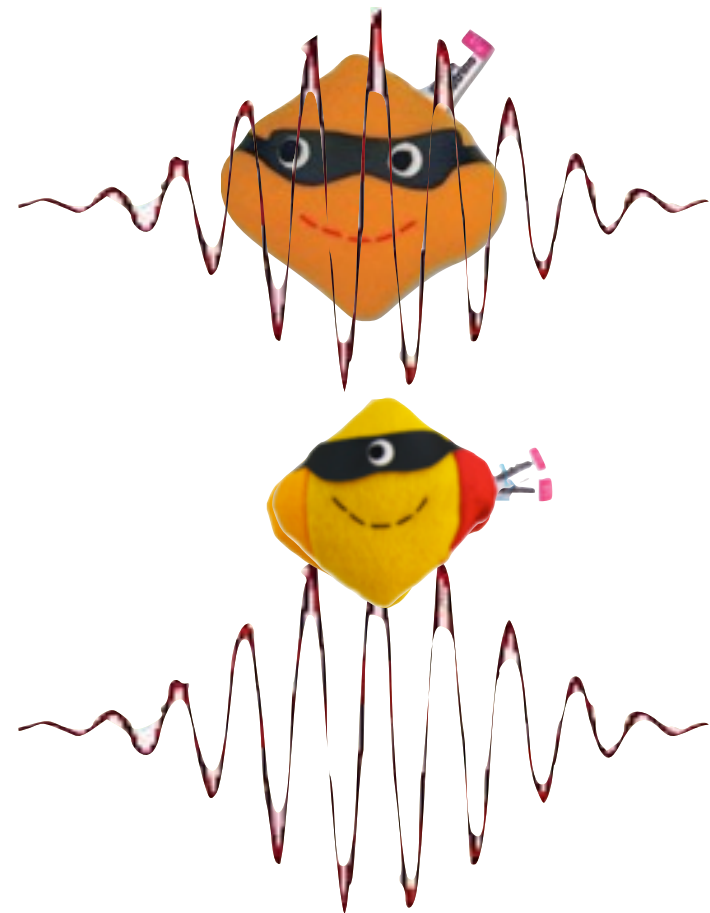


Let's say that we start off with a muon neutrino — this is a flavor eigenstate



Let's make it a muon neutrino from the NuMI beam

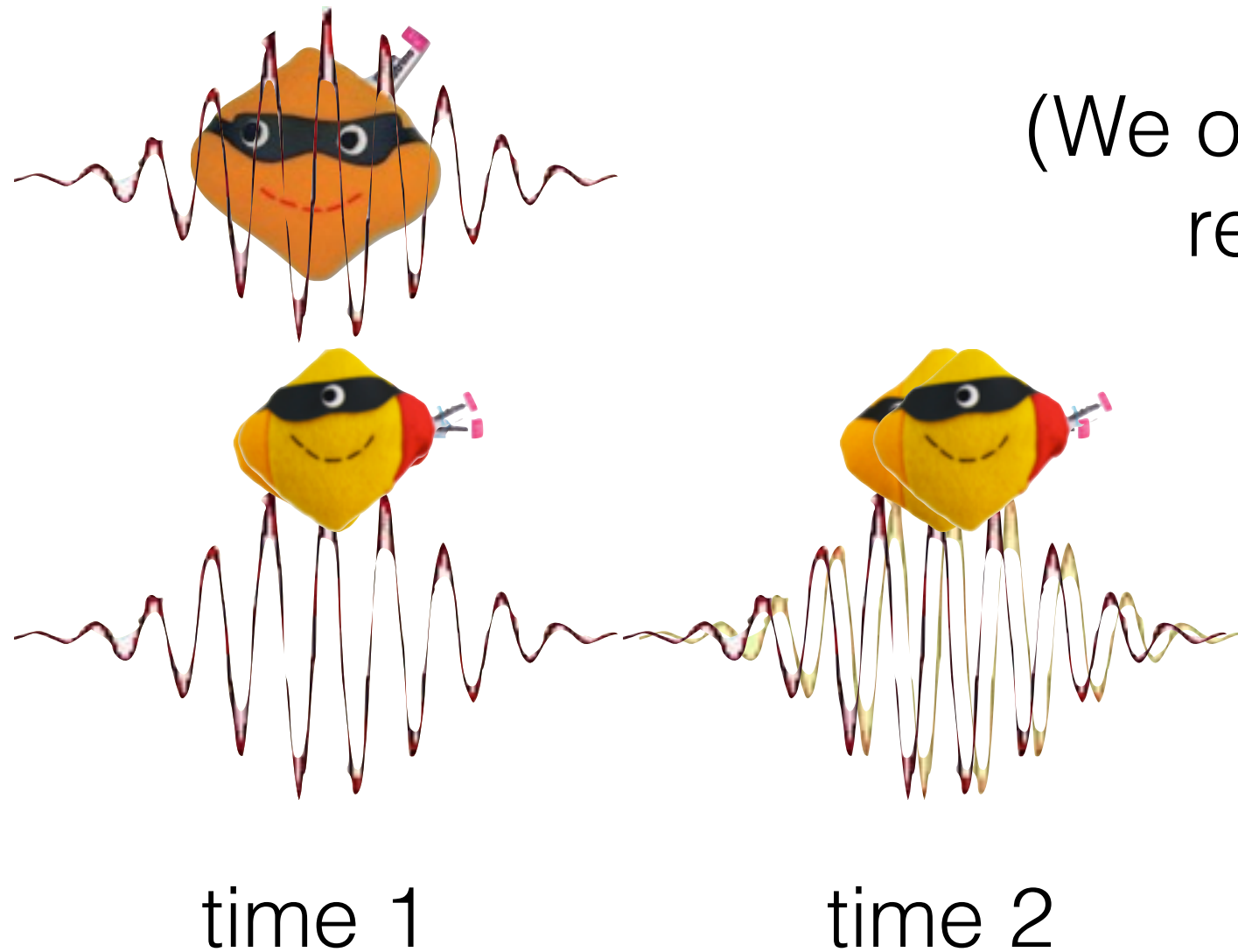
# A flavor state is in a superposition of mass eigenstates.



And particles, remember, act as waves  
— waves can interfere with one another

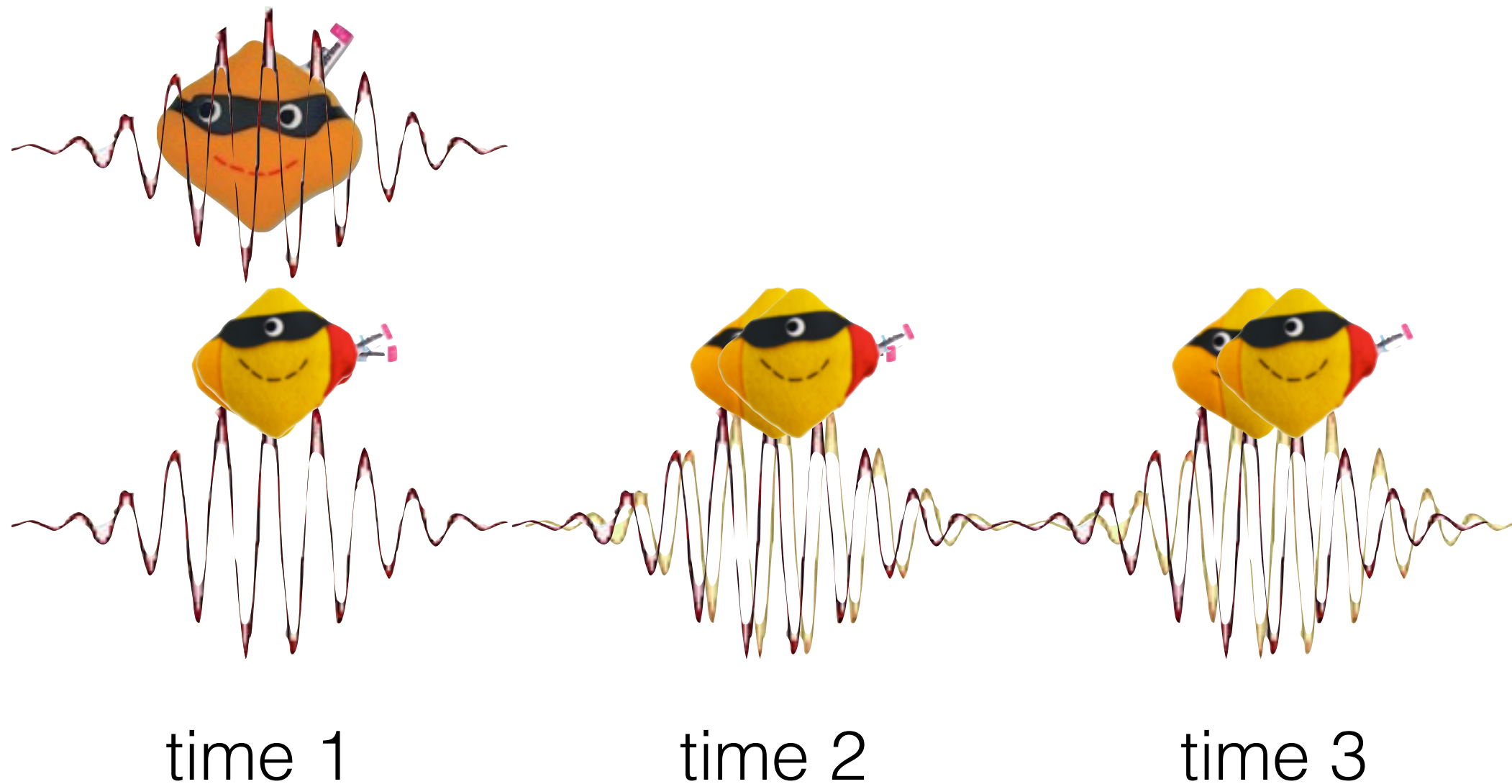
time 1

# As the neutrino moves, the mass states composing it begin to fall out of sync

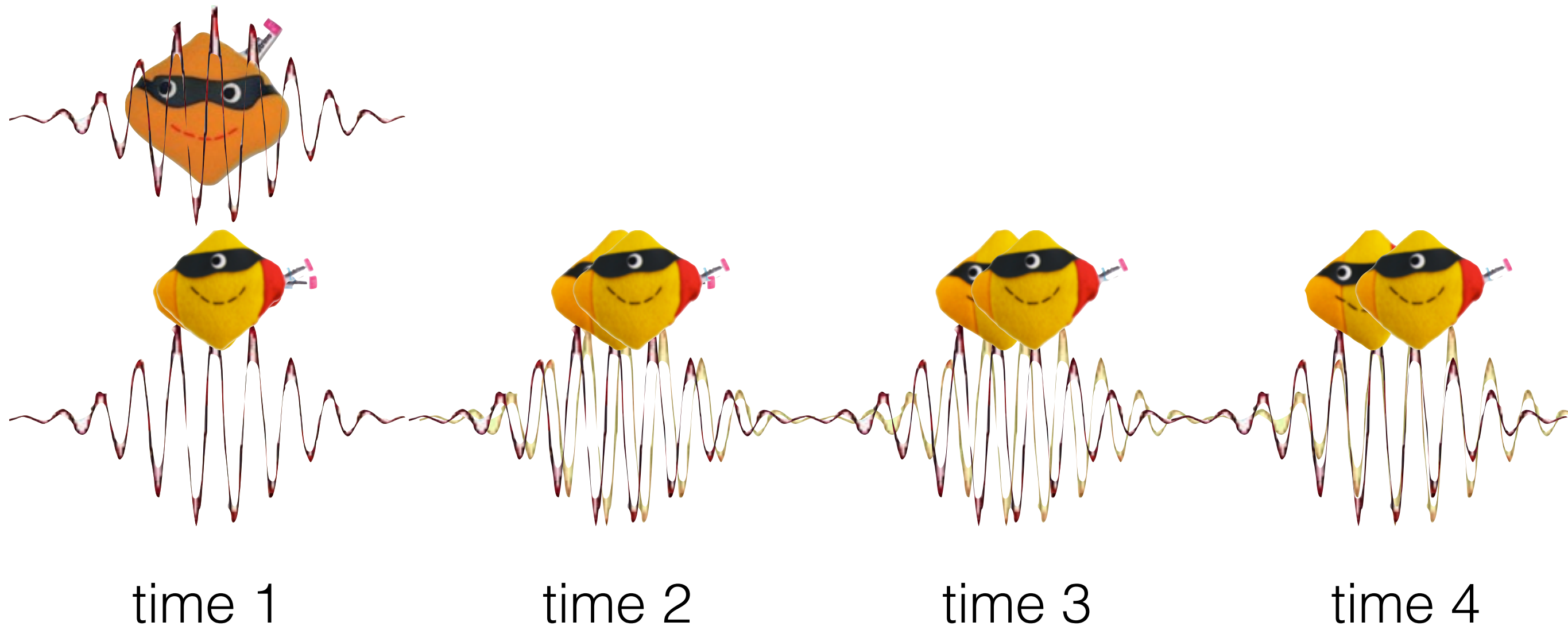




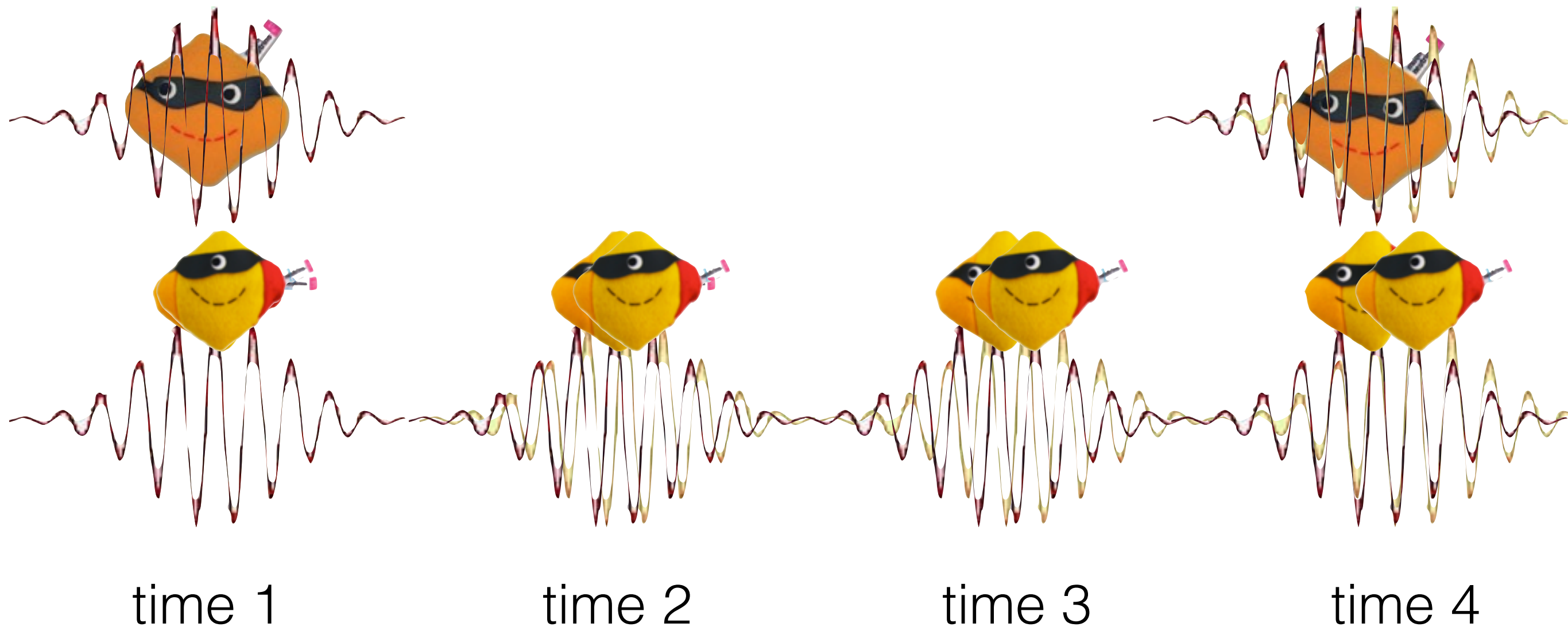
# But as the heavier states lag behind, eventually...



The mass states begin to interfere in a way that looks similar to the way they started

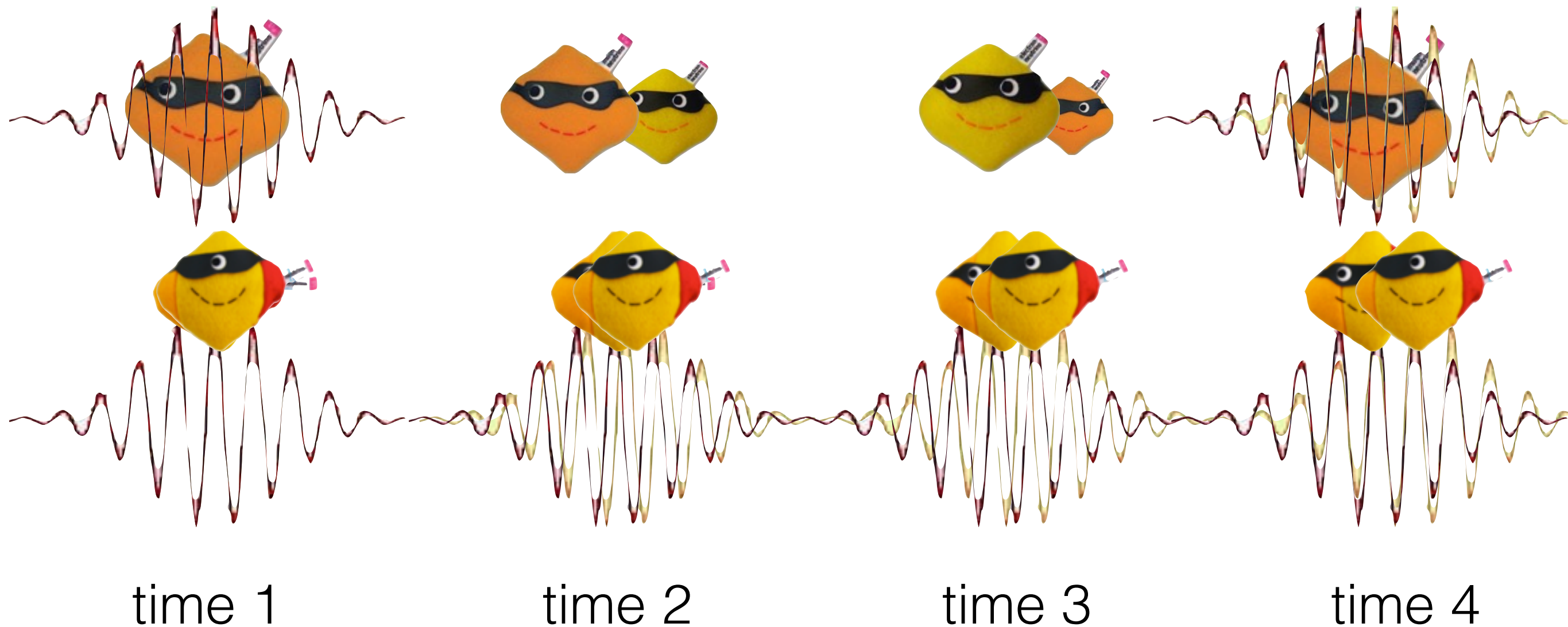


So if the neutrino interacts at time 4, we have a very good chance of observing a muon neutrino again



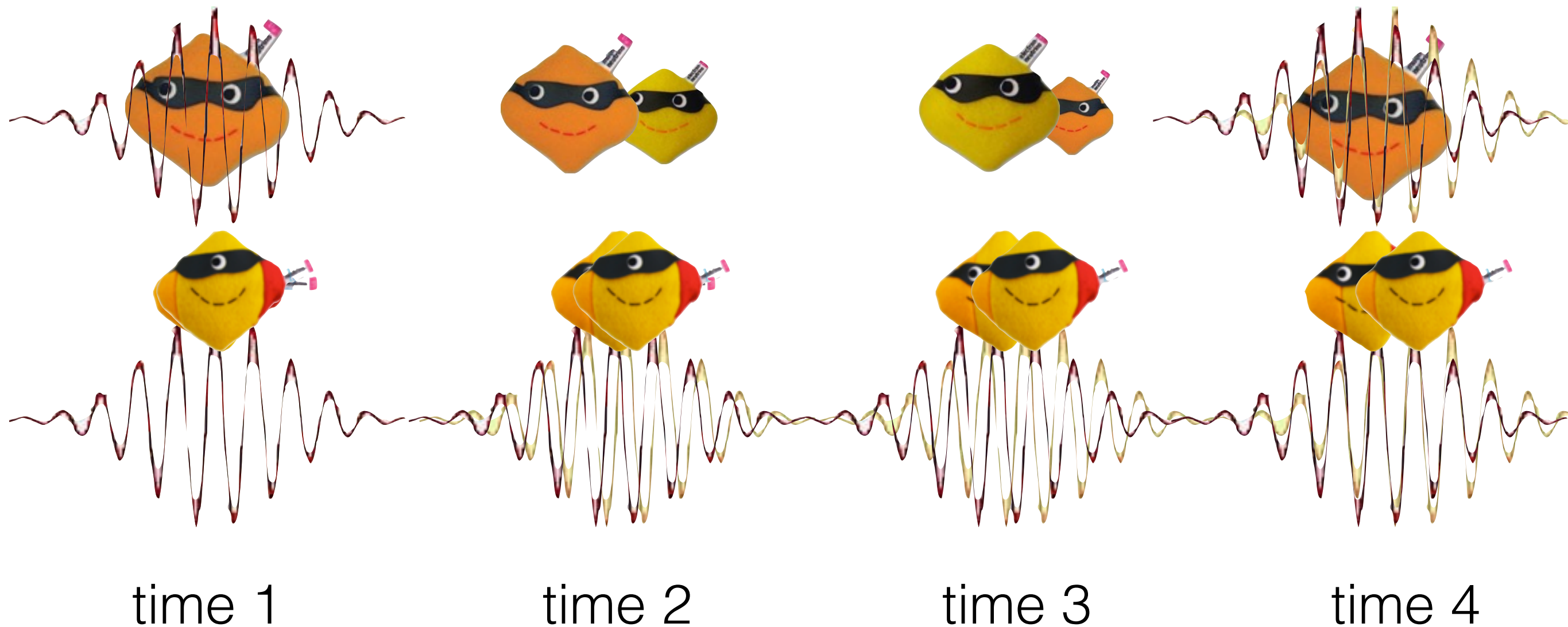


At other times, we might have a better chance of seeing, say, an electron neutrino



\*Don't try to read too much into the wave packets —> plushies translation!  
The plushie distributions shown are ~arbitrary.

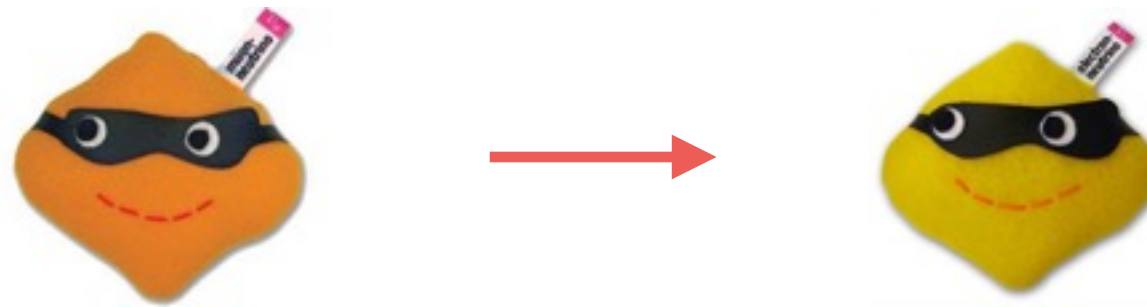
# This is, in essence, neutrino oscillation



Again, in reality, there are three mass eigenstates involved!\*

\*More if there are sterile neutrinos, but we won't talk about them in this talk.

# Neutrino oscillation is described by a whole slew of variables



$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= | U_{\mu 1}^* e^{-im_1^2 L/2E} U_{e1} + U_{\mu 2}^* e^{-im_2^2 L/2E} U_{e2} + U_{\mu 3}^* e^{-im_3^2 L/2E} U_{e3} |^2 \\
 &= | 2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21} |^2 \\
 &\approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} + \delta)} + \sqrt{P_{sol}} |^2.
 \end{aligned}$$

$\delta_{CP}$  **sign( $\Delta m_{32}$ )**



$\theta_{23}$

Equation Source: [S. Parke, "Determining the Neutrino Mass Hierarchy"](#)

These variables usually appear **only in combination**.



# How can we cut these terms apart, and see what nature has to tell us?

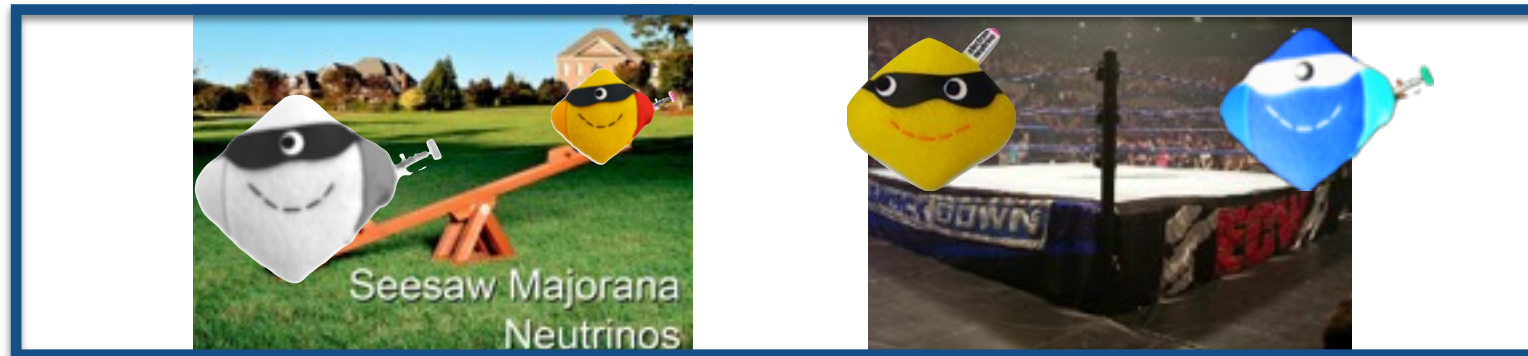


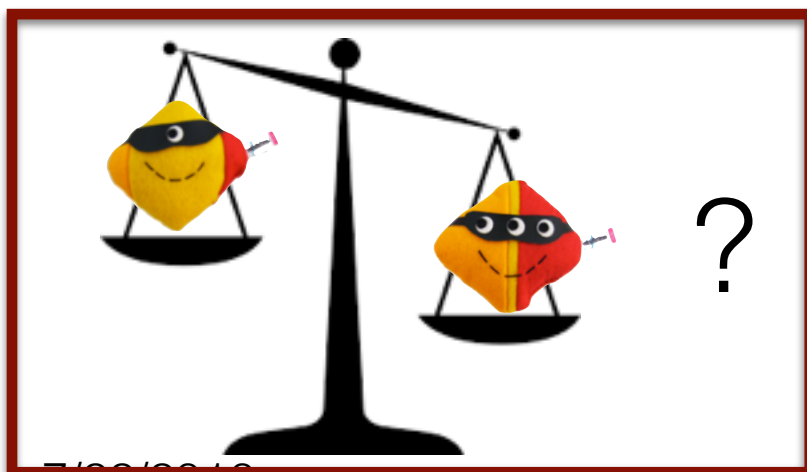
Image (without plushies): [Fermilab Today](#)

Image: [Wikipedia, Wrestling Ring](#)

$\delta_{CP}$



$\theta_{23}$



$\text{sign}(\Delta m_{32})$



Image [Source](#)

# PART II

## Neutrino masses



# We know that there are three neutrino mass states



But there's a lot about these states that is unknown.

For one, the jury's still out on whether the third mass eigenstate is more muon or tau-neutrino like

$$\theta_{23} > 45^\circ$$



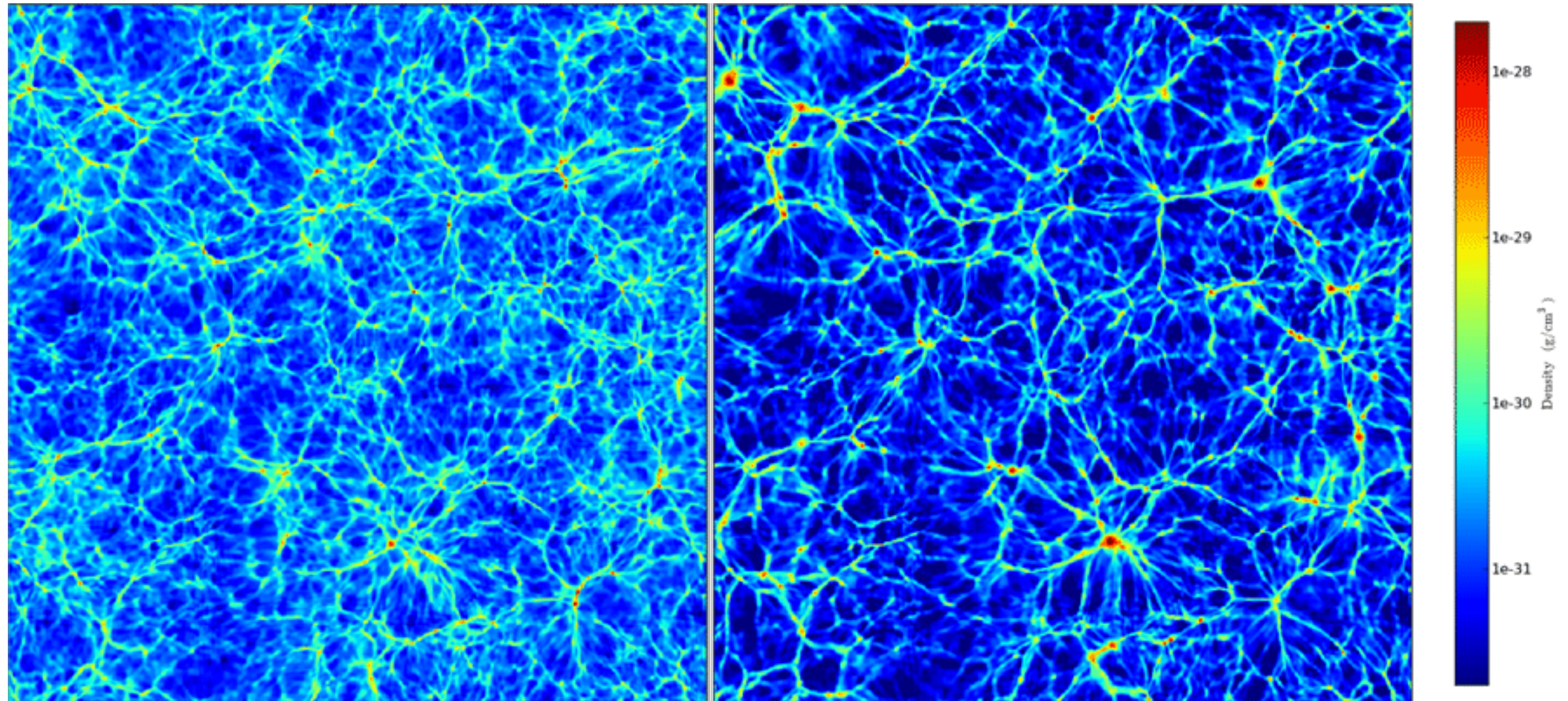
$$\theta_{23} < 45^\circ$$

We also are not even sure what the masses of these states are!



# An upper limit on the **sum of the three neutrino masses** is estimated at **$< 0.3 \text{ eV}$**

Image: Viewpoint: Galaxies weight in on neutrinos

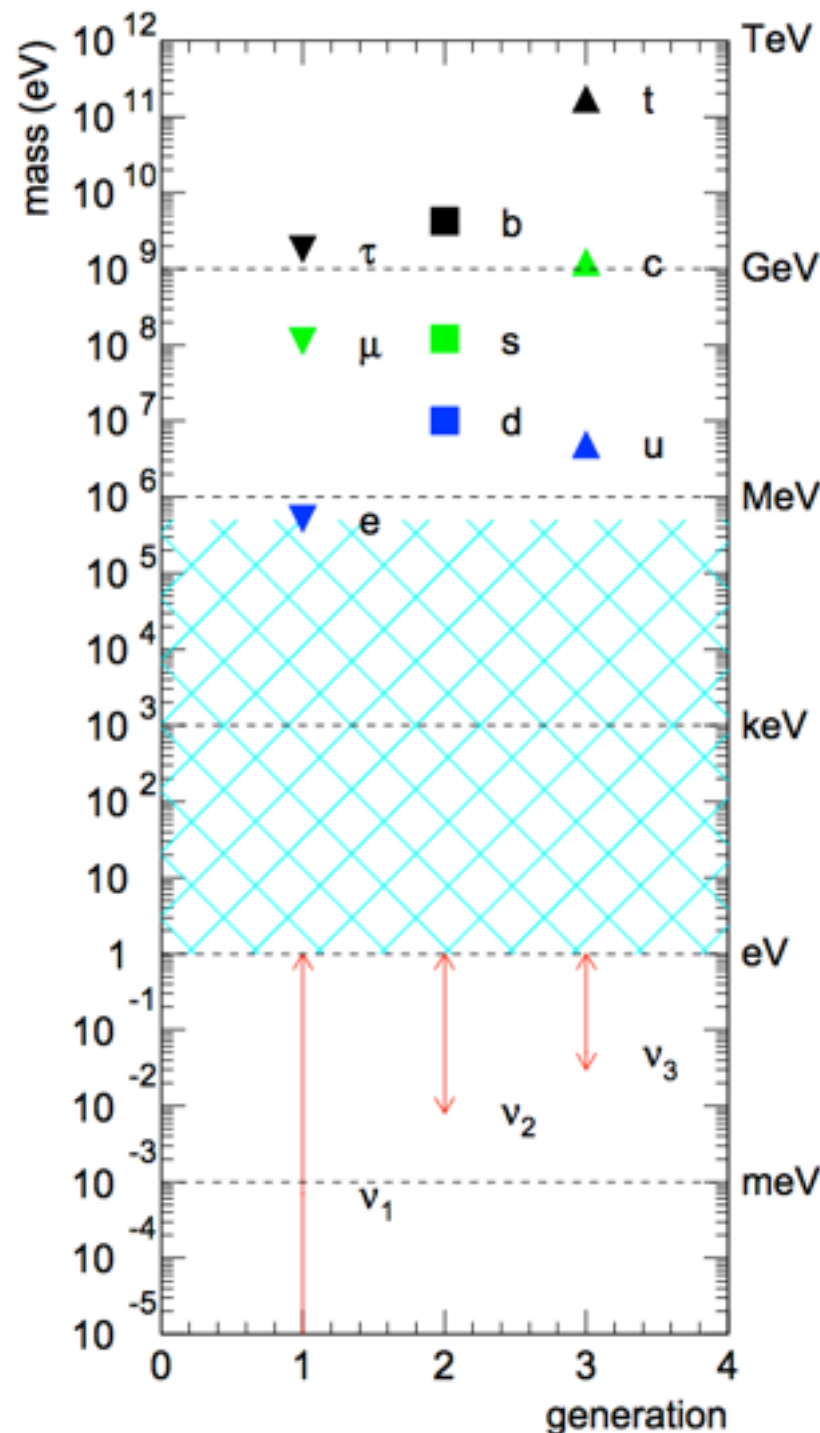


Massive Neutrinos

Massless Neutrinos

This was determined by exploring the effect of neutrino mass on structure formation in the early universe

# Neutrino masses are oddly small



The basic Standard Model predicts that they ought to be *massless*.

But neutrinos *have* mass...  
six orders of magnitude smaller than the other elementary particles!

*Do neutrinos acquire mass in the same way as other particles?*



# Quarks and charged leptons get their mass from a “Yukawa” coupling to the Higgs field

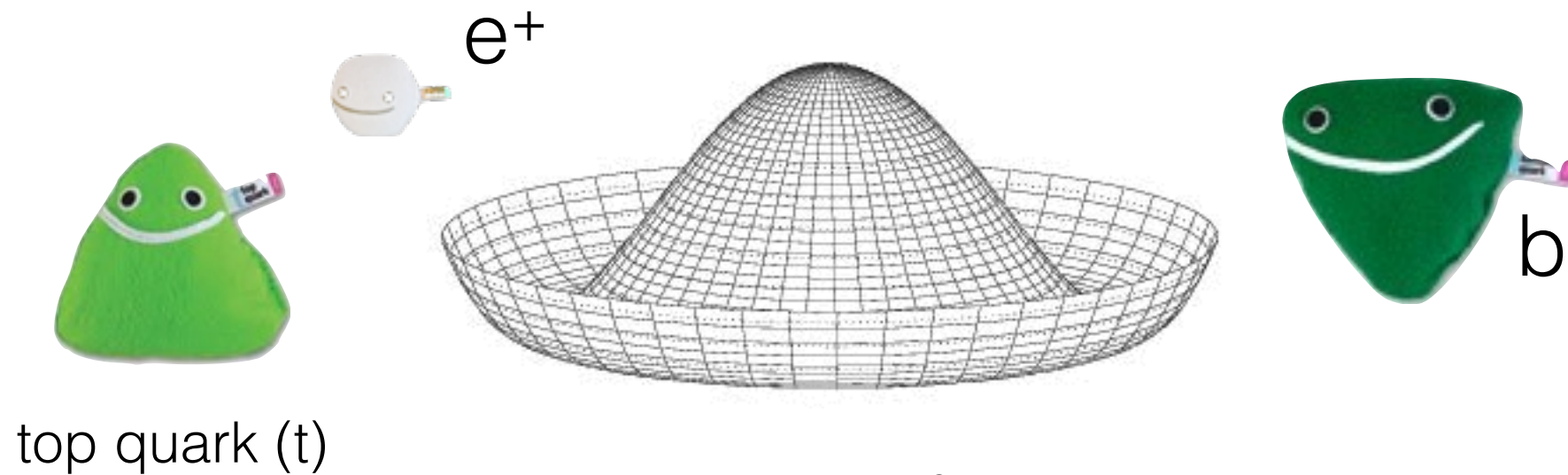
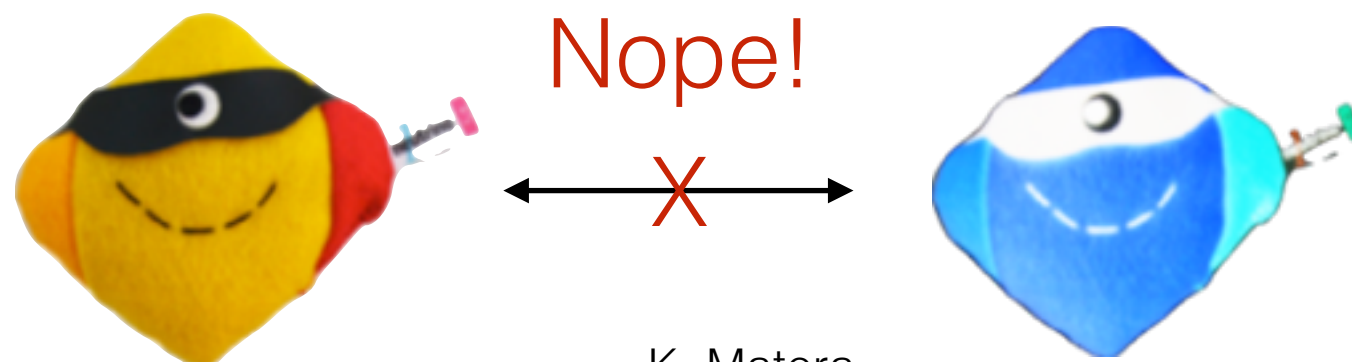


Image: [Physics Master Classes](#)

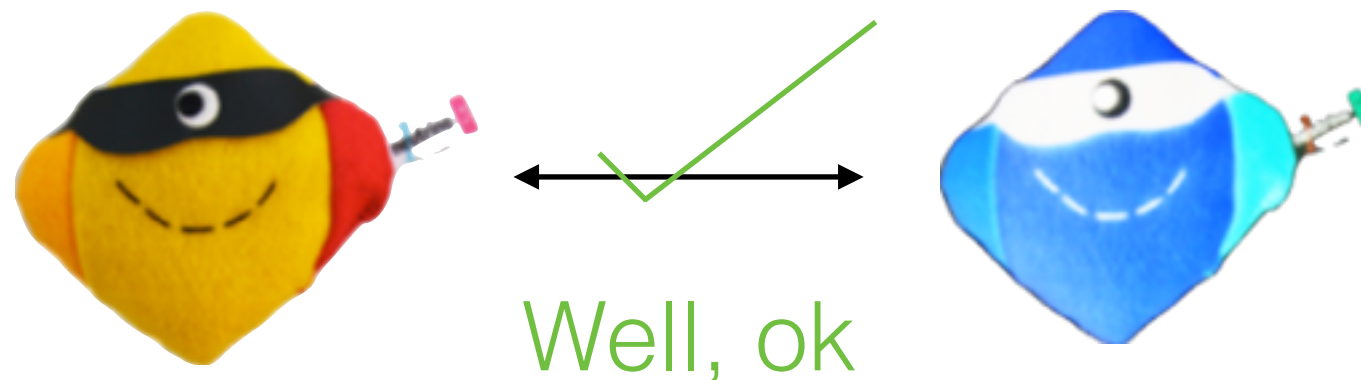
This is called a “Dirac” mass, and has the effect that **Dirac neutrinos and antineutrinos do not “mix”**





There are different mass generators that *do* allow neutrinos and anti-neutrinos to mix.

They **either involve the Higgs field in a different way, or not at all!**



These are called “Majorana” masses, and neutrinos with such masses are **Majorana neutrinos**

A **Majorana neutrino** would be **its own antiparticle**.

# Given all of this — do neutrino masses follow expectation?\*

The electron is lighter than the muon, is lighter than the tau...



So we might vaguely 'expect' that neutrinos follow a similar pattern.

In that case, we anticipate:



Mostly electron-neutrino



~ equal contributions from all flavors



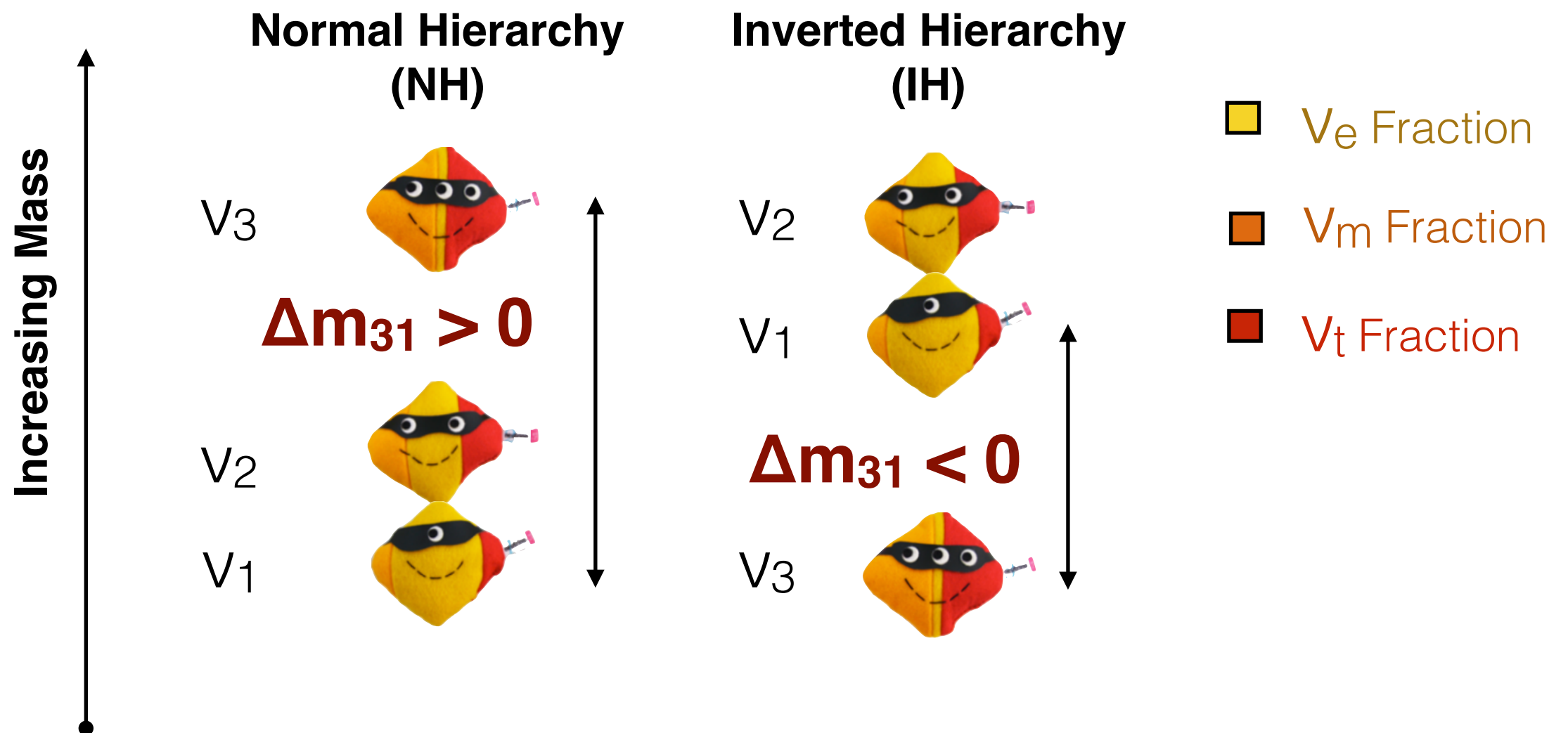
Mostly muon and tau-neutrino

This is the **normal hierarchy**

*\*The hierarchy does not directly imply Dirac or Majorana neutrinos!*

# We have not yet determined whether the normal hierarchy holds

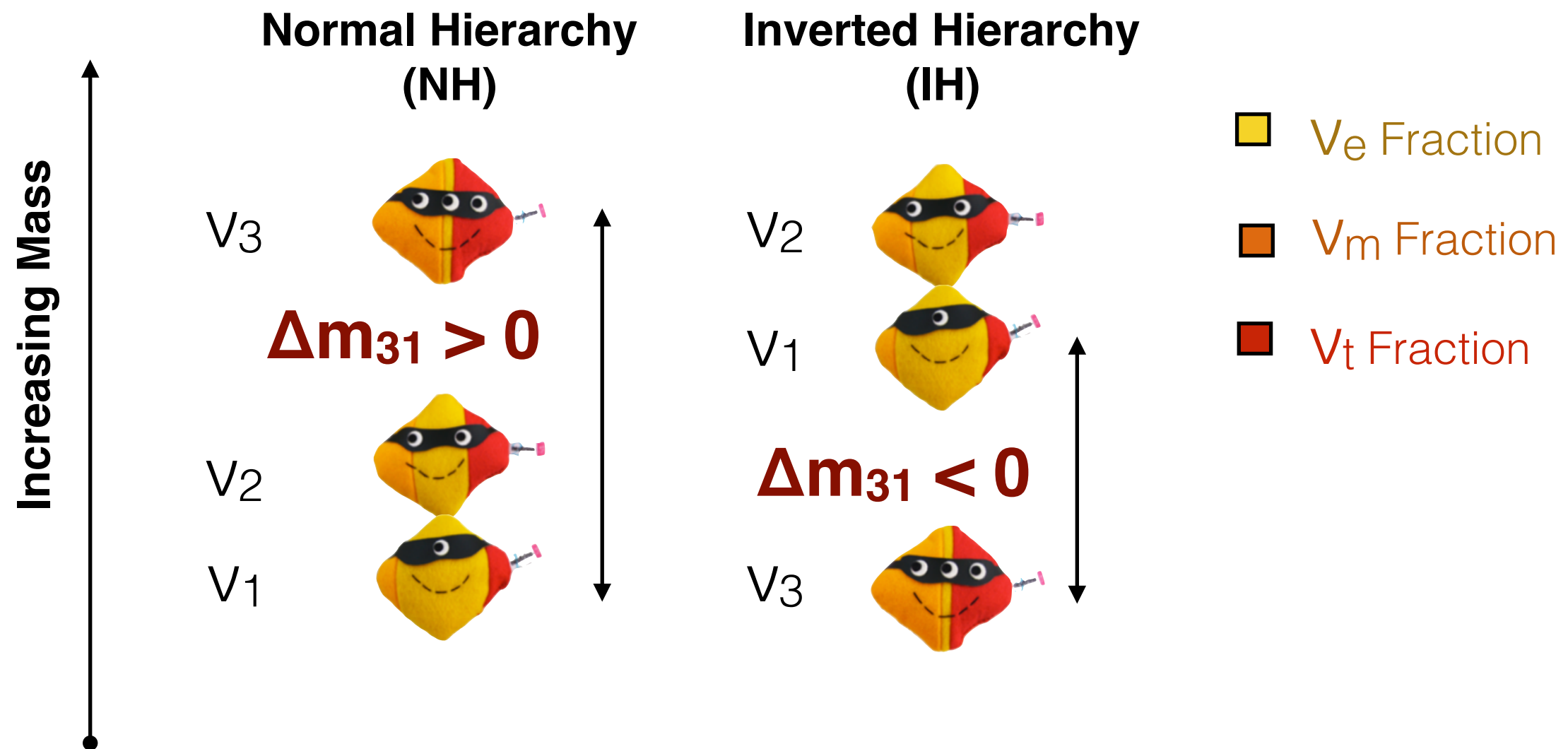
We have only determined that  $v_2 > v_1$



If our “expectation” is flipped,  
we are looking at the **inverted hierarchy**.



# This is the Mass Hierarchy Problem



Solving it would help to untangle our knot.

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= |U_{\mu 1}^* e^{-im_1^2 L/2E} U_{e1} + U_{\mu 2}^* e^{-im_2^2 L/2E} U_{e2} + U_{\mu 3}^* e^{-im_3^2 L/2E} U_{e3}|^2 \\
 &= |2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21}|^2 \\
 &\approx |\sqrt{P_{atm}} e^{-i(\Delta_{32} + \delta)} + \sqrt{P_{sol}}|^2.
 \end{aligned}$$

Equation Source: S. Parke, "Determining the Neutrino Mass Hierarchy"

$\delta_{CP}$  **sign( $\Delta m_{32}$ )**



$\theta_{23}$

# Finding a NH or IH will affect how we answer the Majorana vs Dirac question

This might be done by looking for  
neutrino-less double-beta decay ( $0\nu\beta\beta$ )

neutron (n)



proton (p)

Consider an unstable nucleus at rest.

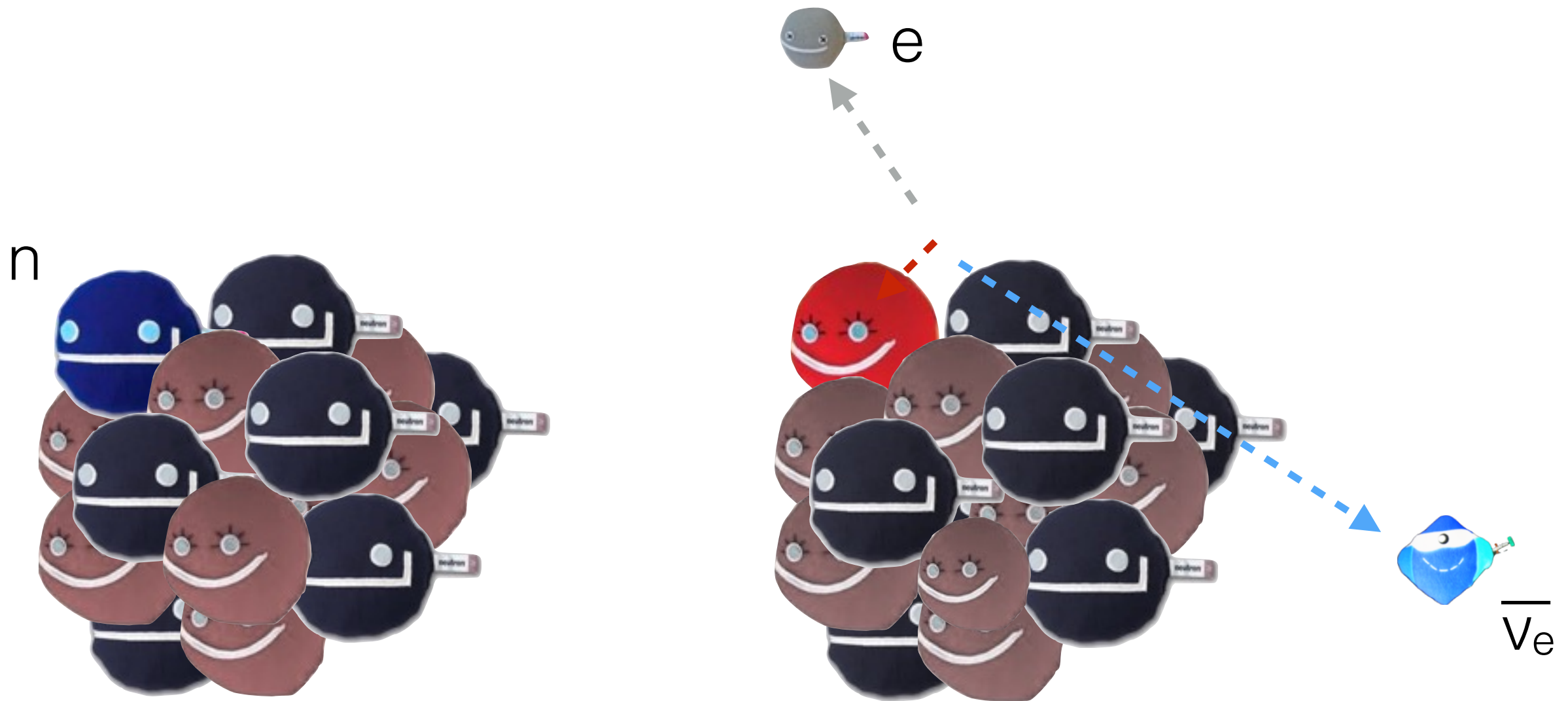
# Neutrons can decay...

n

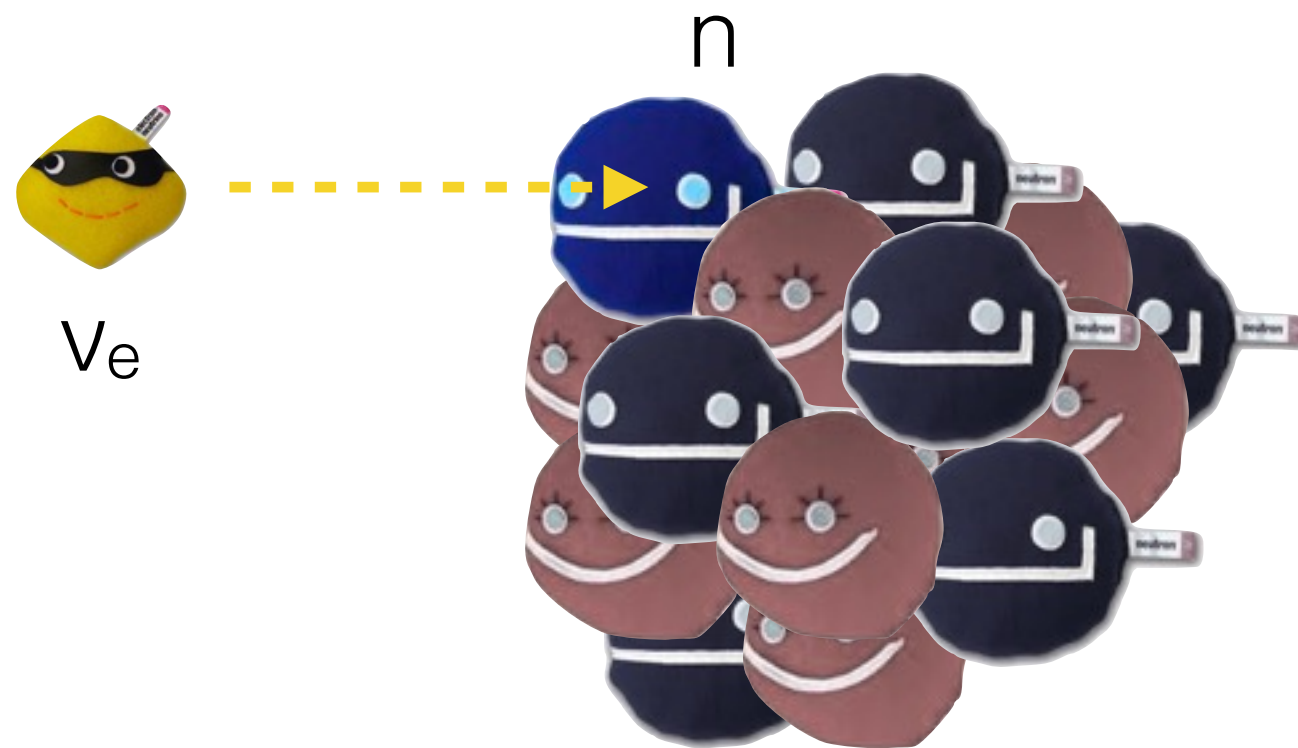




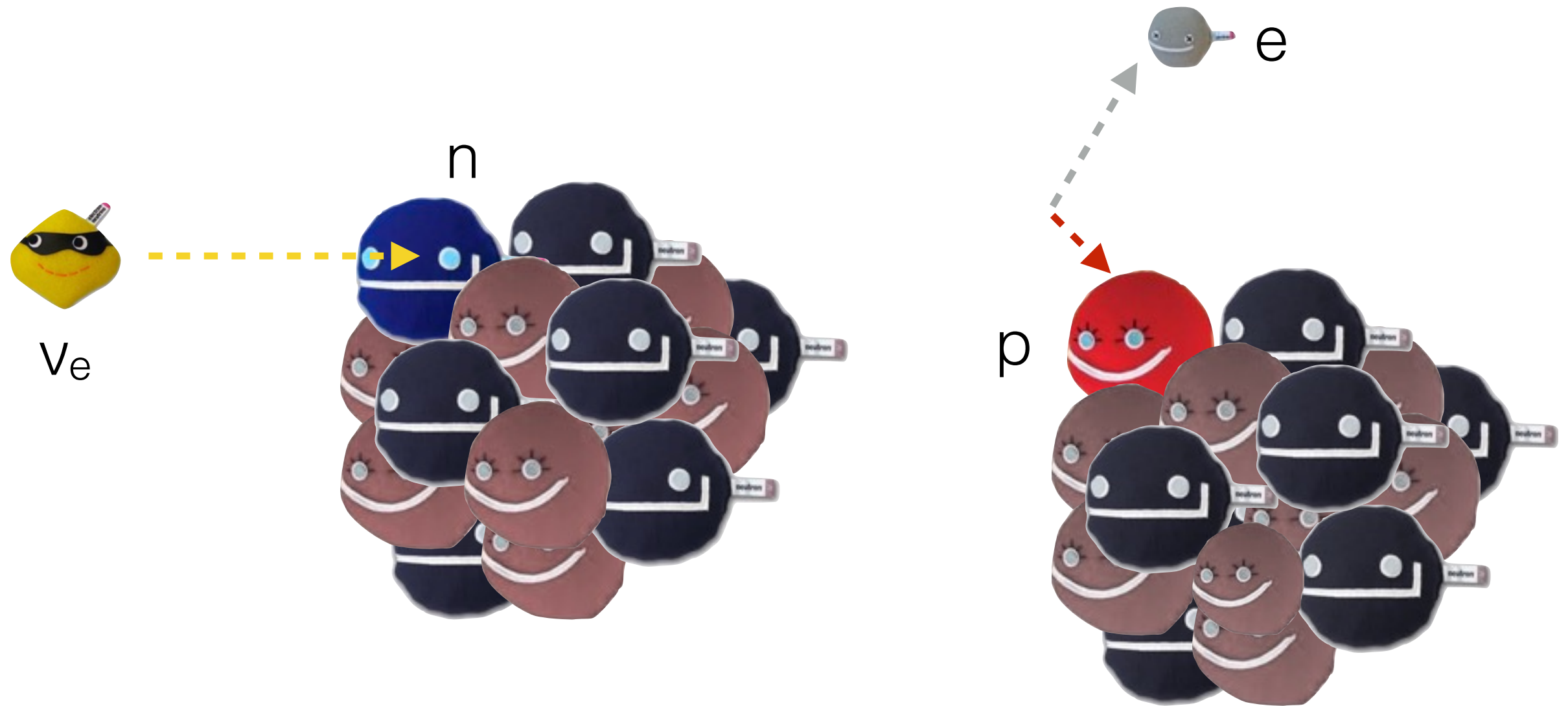
...into protons, with the release  
of an anti-neutrino and an electron



Also recall that an neutrino can  
convert a neutron into a proton



Also recall that an neutrino can convert a neutron into a proton



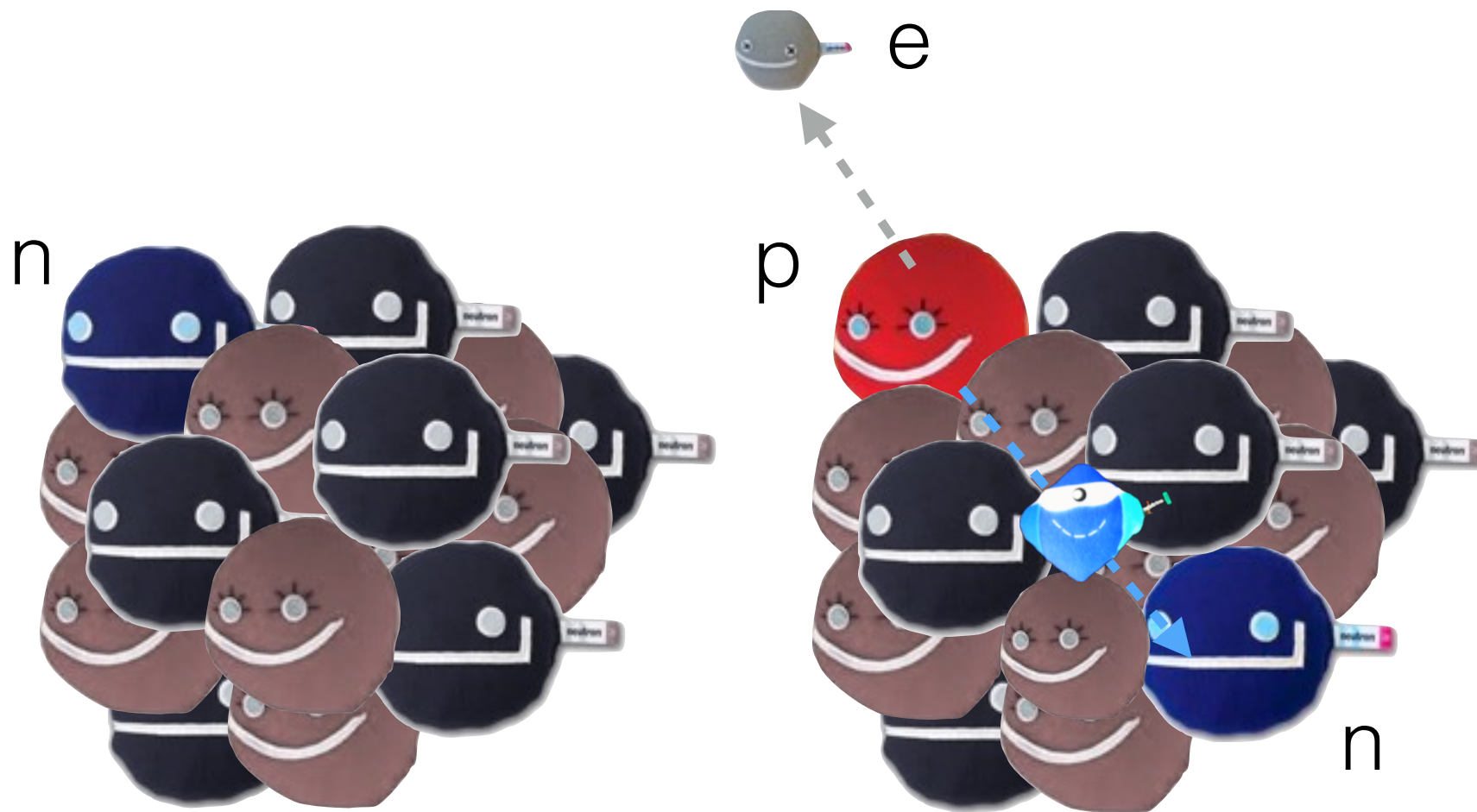
An electron neutrino is 'flipped' into an electron in the process



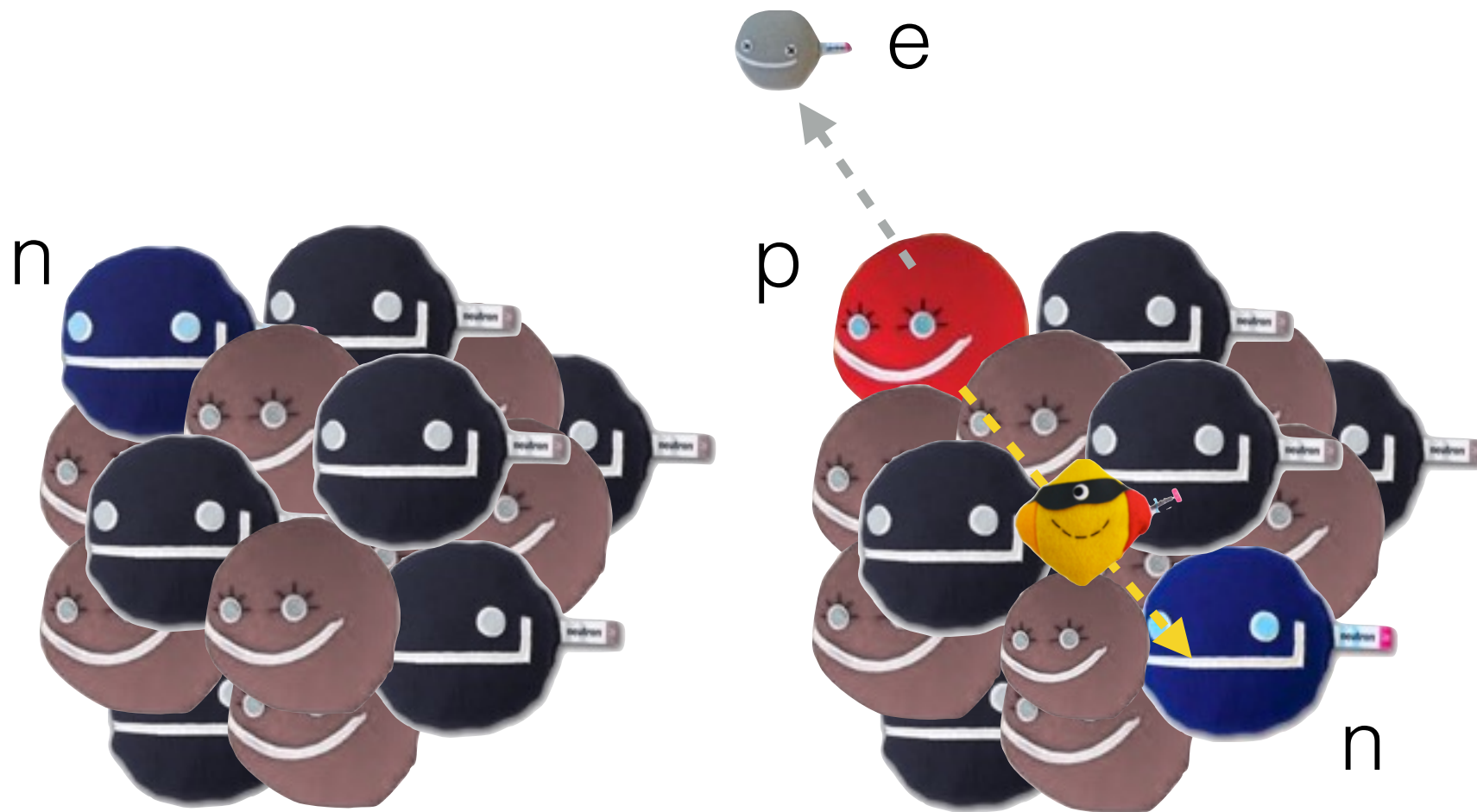
But if a neutrino is its own antineutrino, then we might see...



That the **anti-neutrino** produced by  
neutron decay...

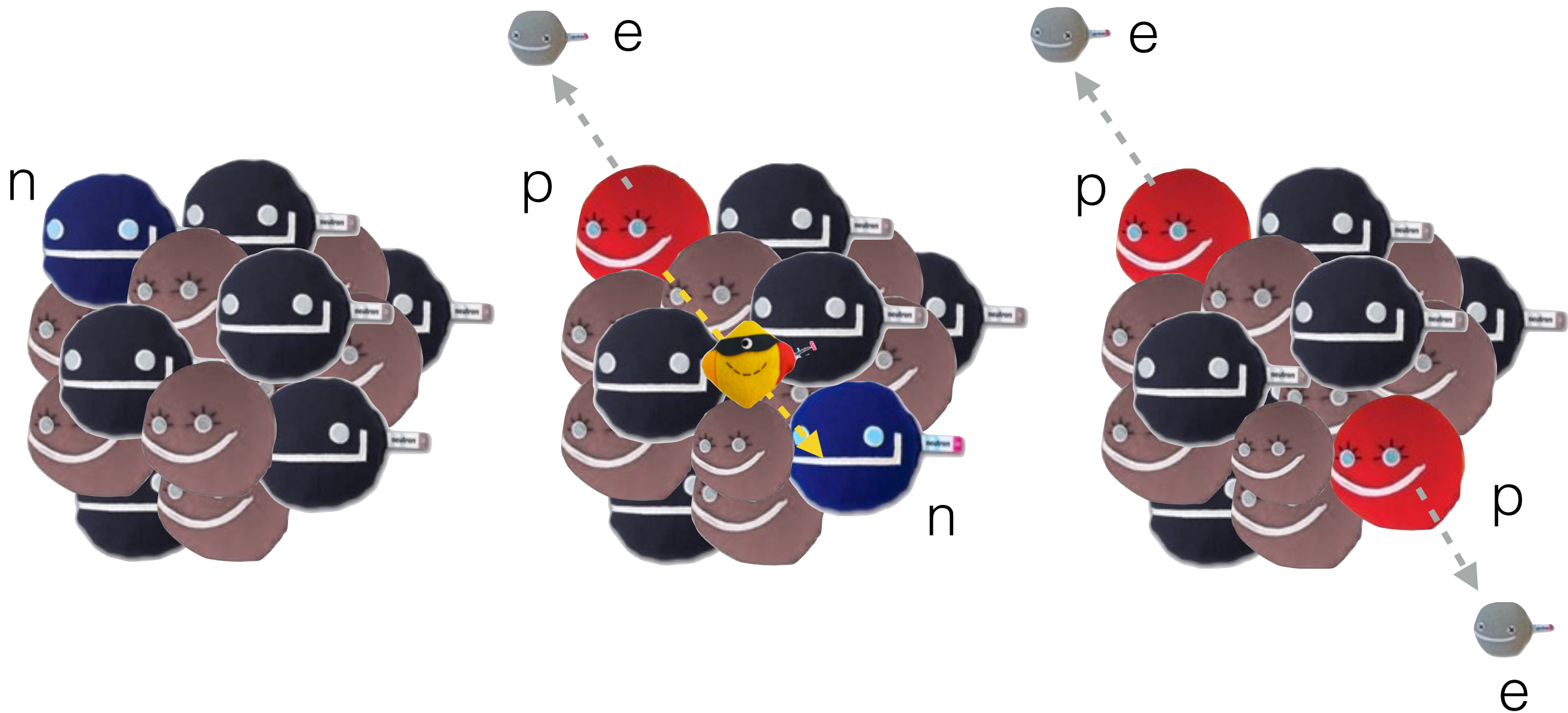


Will interact **as a neutrino** with another neutron in the same nucleus

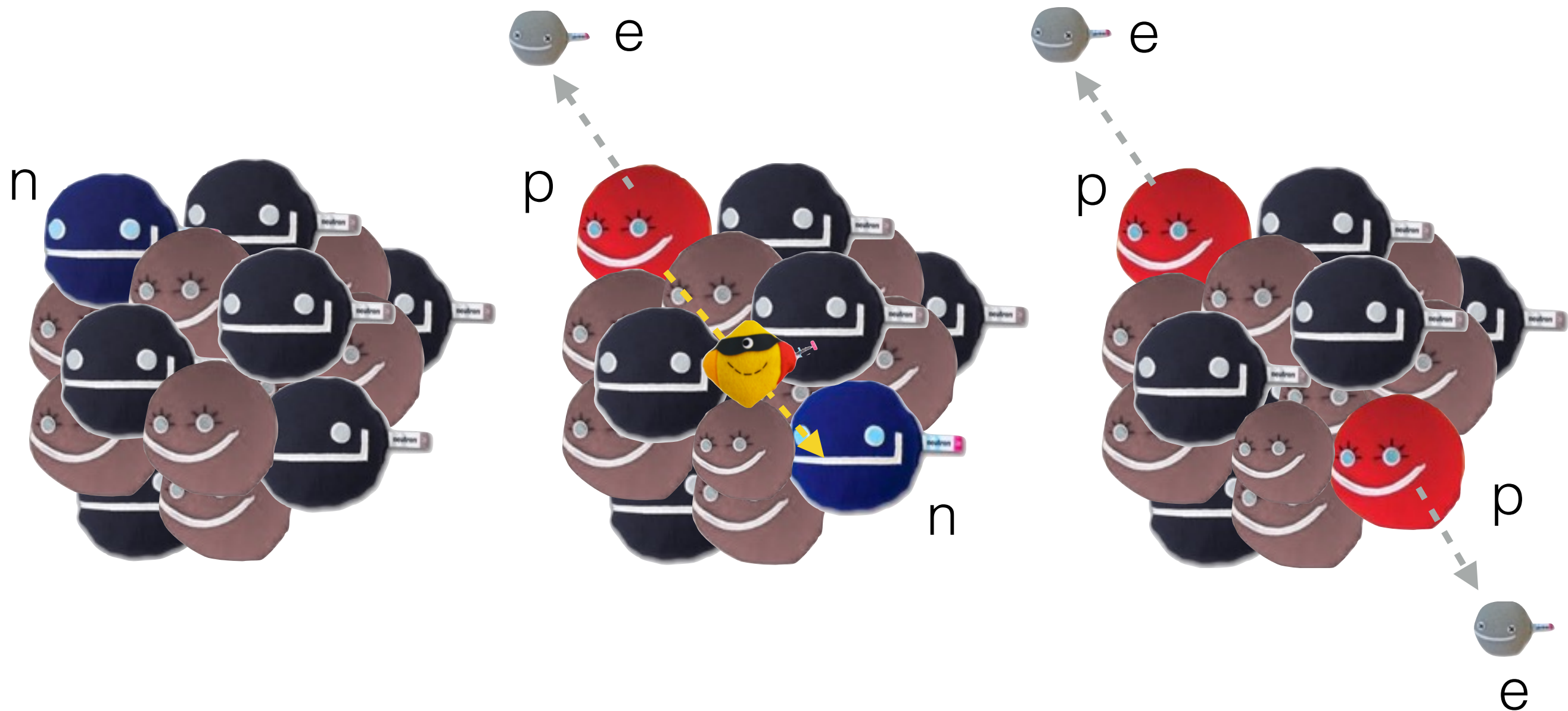




# Producing two beta (electron) particles, but *no neutrinos*...

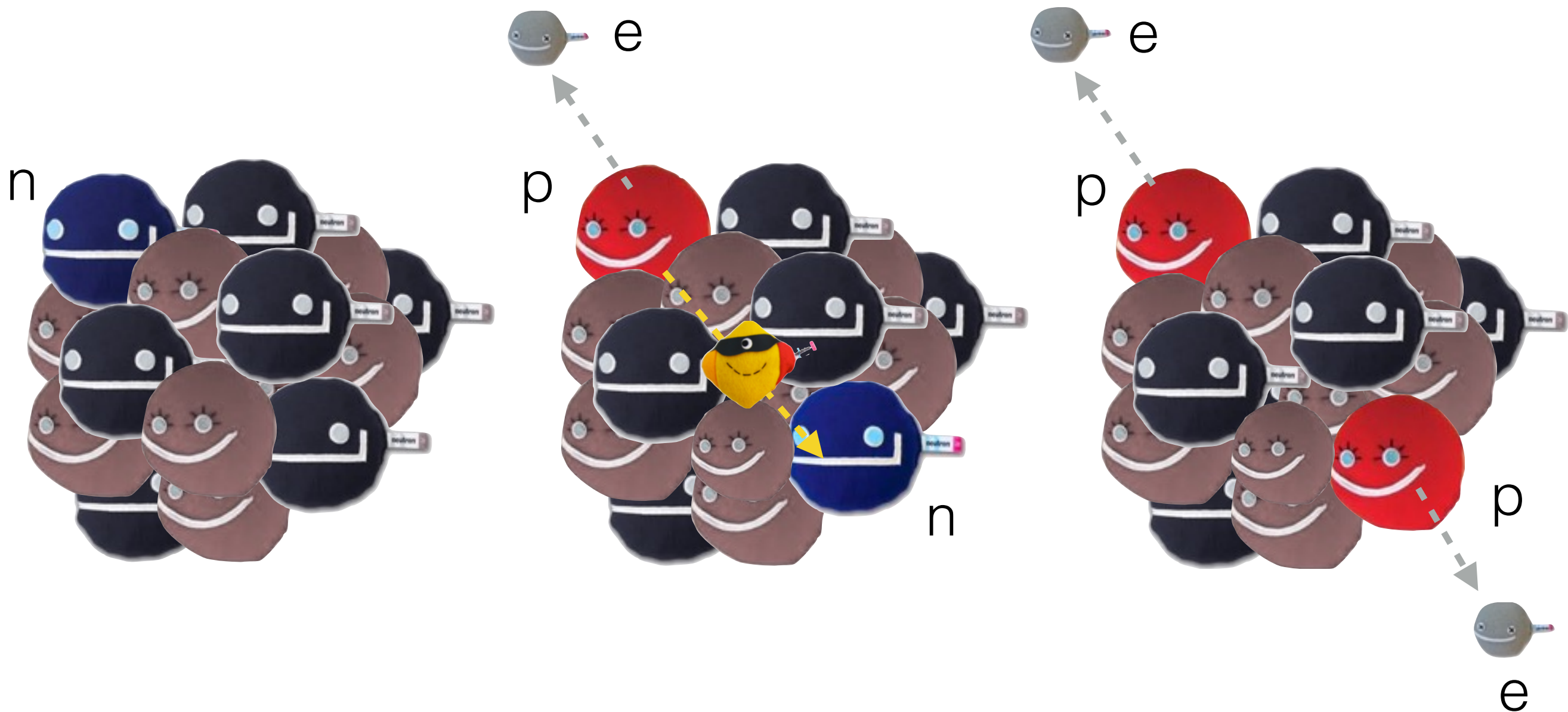


# This is called neutrino-less double beta decay ( $0\nu\beta\beta$ )



The rate of  $0\nu\beta\beta$  decay is proportional to the value of  $m_{ee}$ .

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The rate of  $0\nu\beta\beta$  decay is proportional to the value of  $m_{ee}$ .  
The value of  $m_{ee}$  depends on the mass hierarchy.



# A **direct mass measurement** could eliminate the inverted hierarchy option



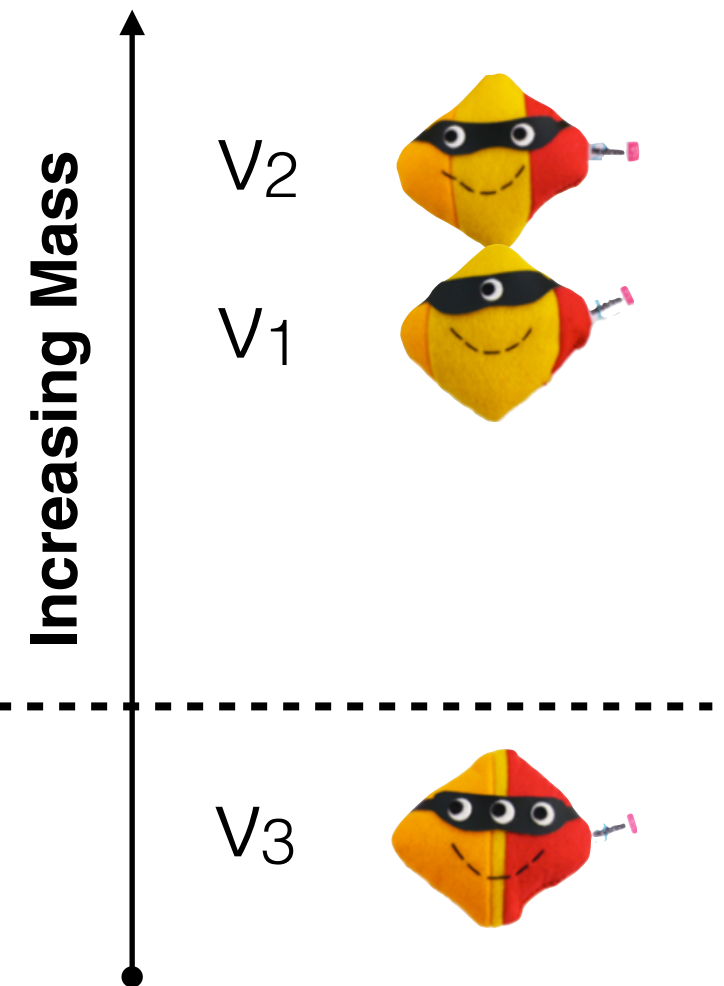
Say we found  $m(\nu_1) = .03 \text{ eV}$

0.03 eV —————

We know  $\text{mag}(\Delta m_{31}) \sim .05 \text{ eV}$

0.00 eV - - - - -

-0.02 eV —————



This would put  $m(\nu_3) < 0$

And so  $m(\nu_1) > m(\Delta m_{31})$   
would effectively rule out the **inverted hierarchy**.

# What if a neutrino has both a Majorana *and* a Yukawa mass term?

This leads to neutrinos with two distinct masses; if one is very large, the other must be very small.

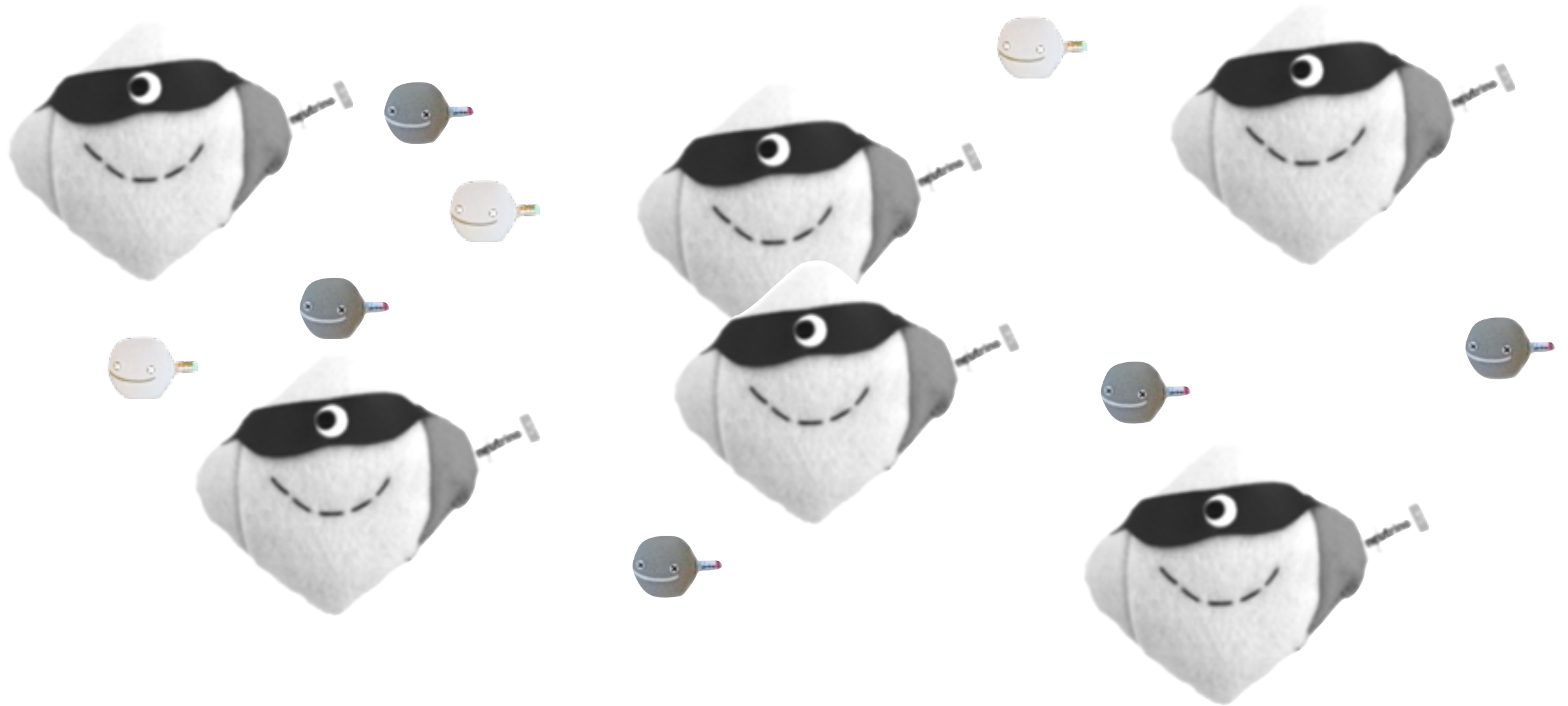


Image (without plushies): [Fermilab Today](#)

$$M(\text{yellow}) \propto \frac{1}{M(\text{white})}$$

This is called the See-Saw mechanism. It predicts that the light neutrinos we see have heavy counterparts.

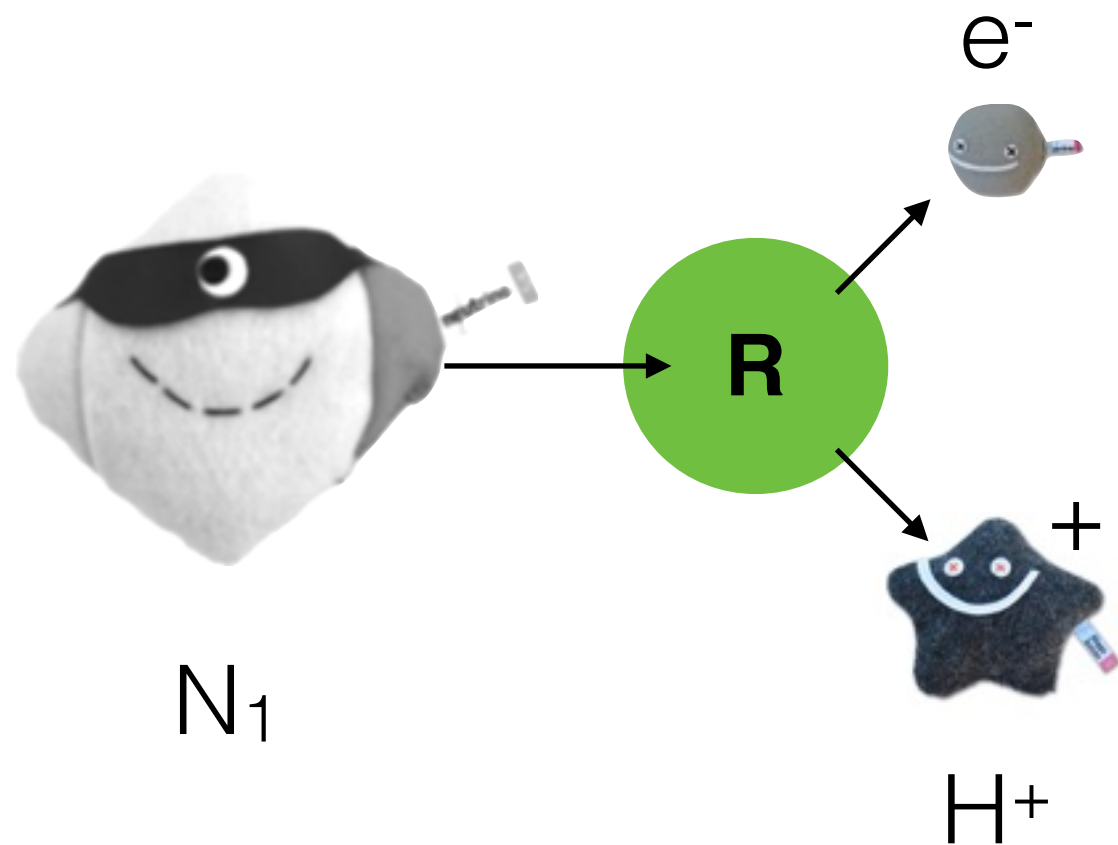
In the early universe, matter and antimatter were produced in equal abundance



This included many heavy neutrinos

# But the neutrino sector includes a CP violating mechanism

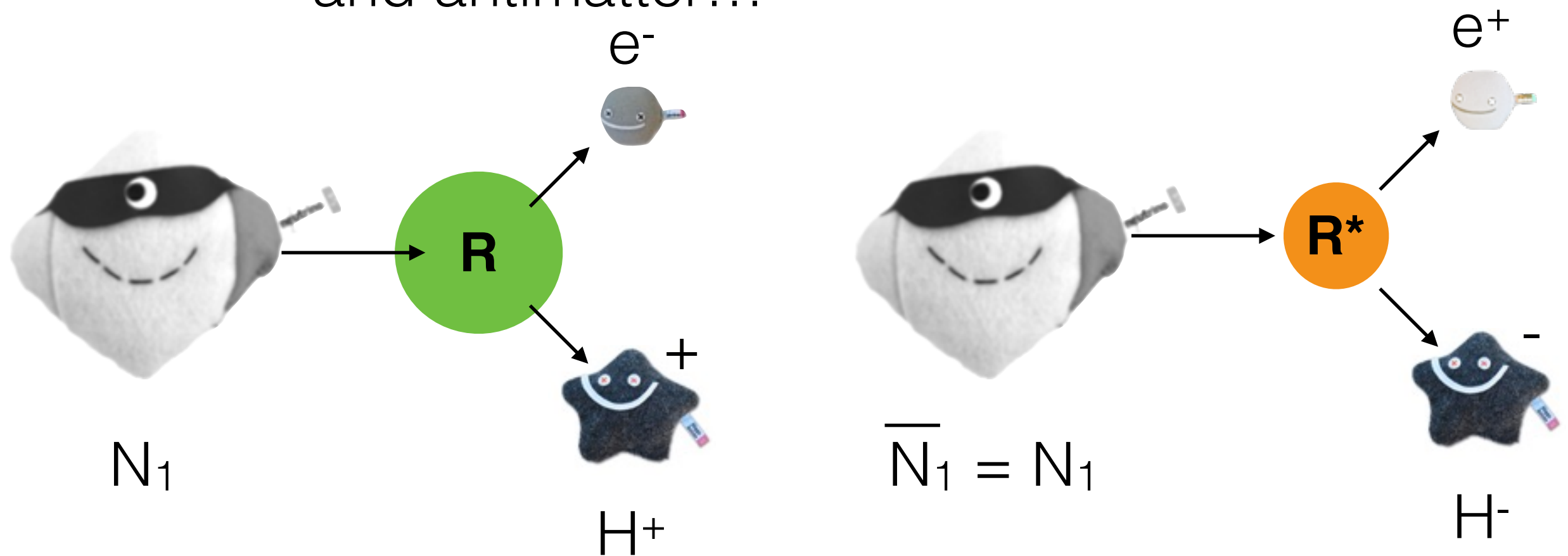
In interactions / decays, CP violation allows matter...





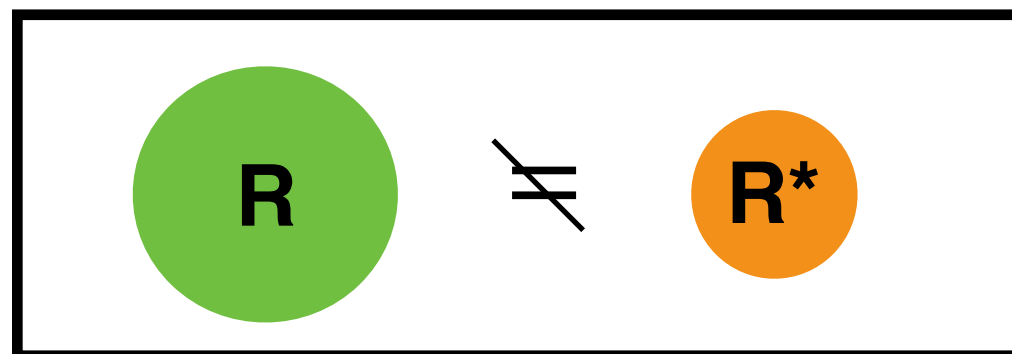
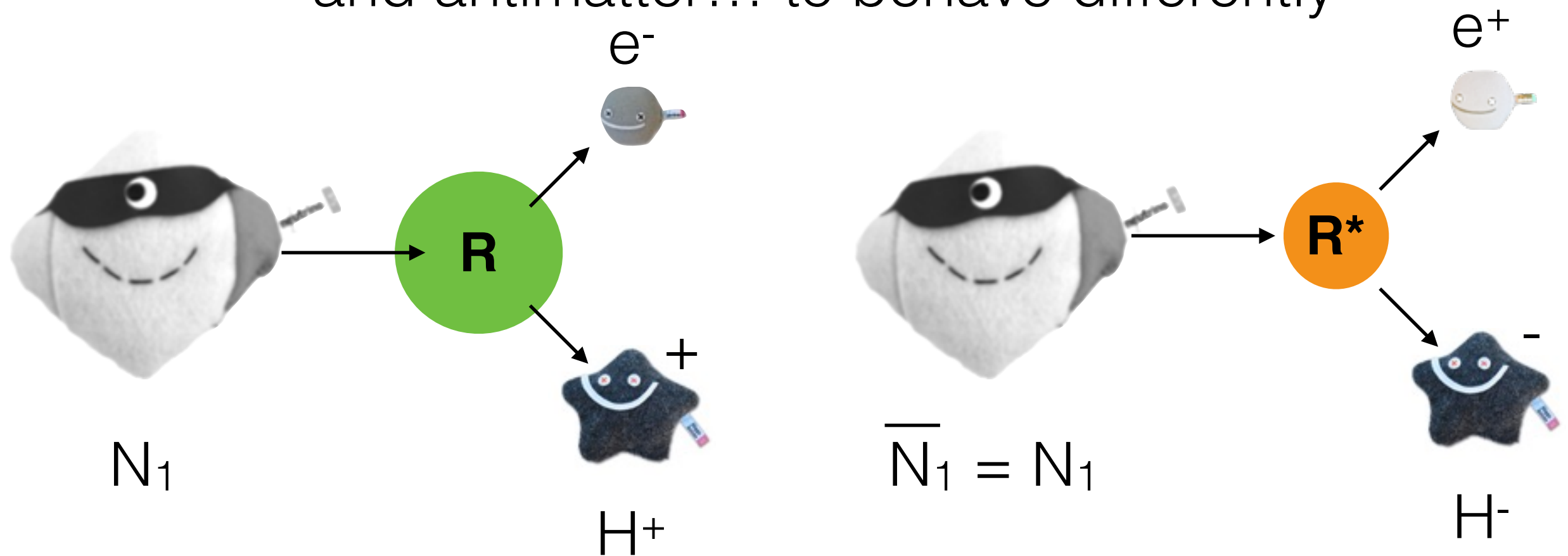
# But the neutrino sector includes a CP violating mechanism

In interactions / decays, CP violation allows matter... and antimatter...



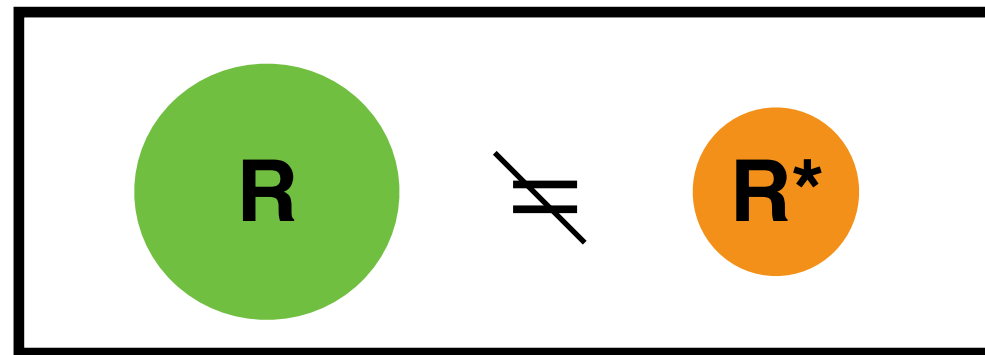
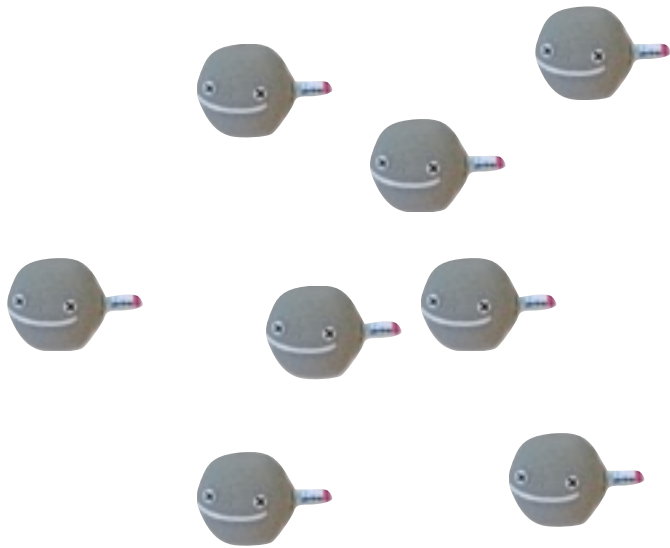
# But the neutrino sector includes a CP violating mechanism

In interactions / decays, CP violation allows matter... and antimatter... to behave differently



This imbalance in the production of leptons and anti-leptons is called **Leptogenesis**

flock of  $e^-$

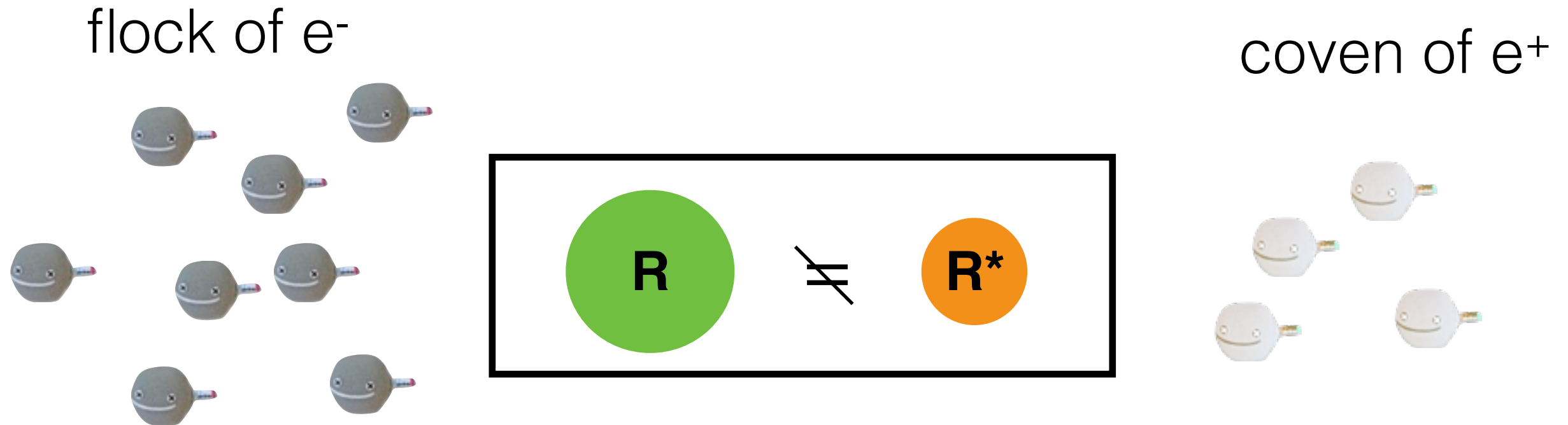


coven of  $e^+$



And it could explain why the universe has an excess of matter over antimatter

This imbalance in the production of leptons and anti-leptons is called **Leptogenesis**

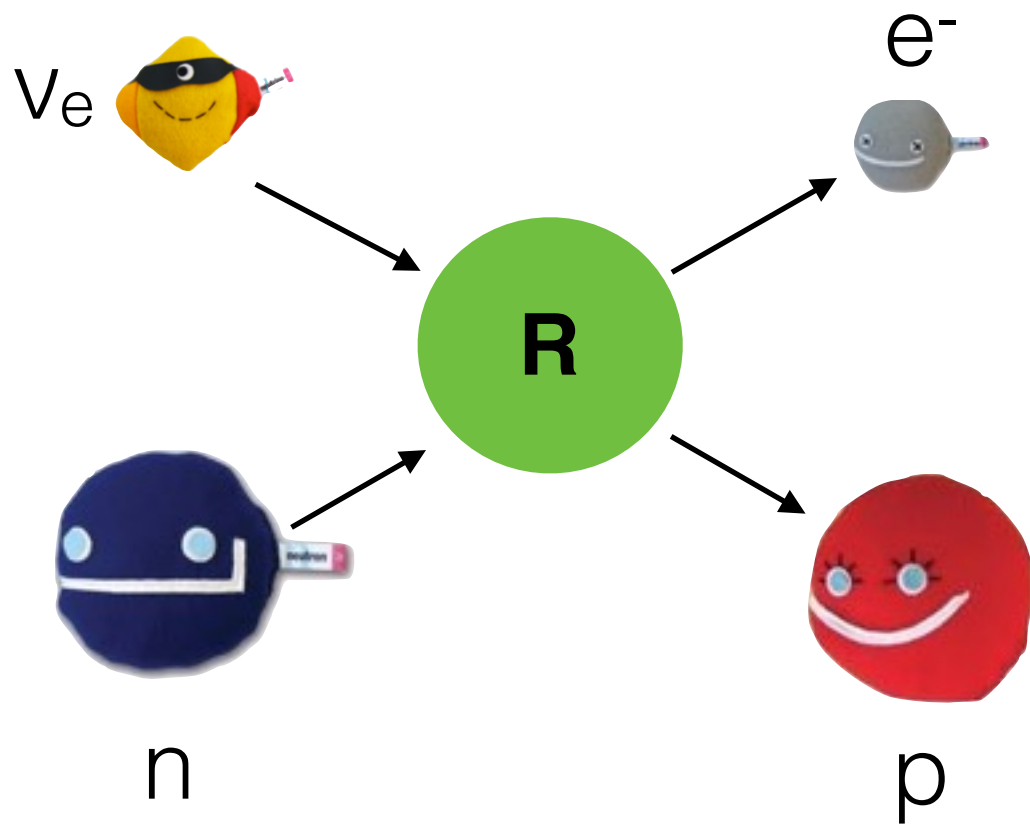


And it could explain why the universe has an excess of matter over antimatter

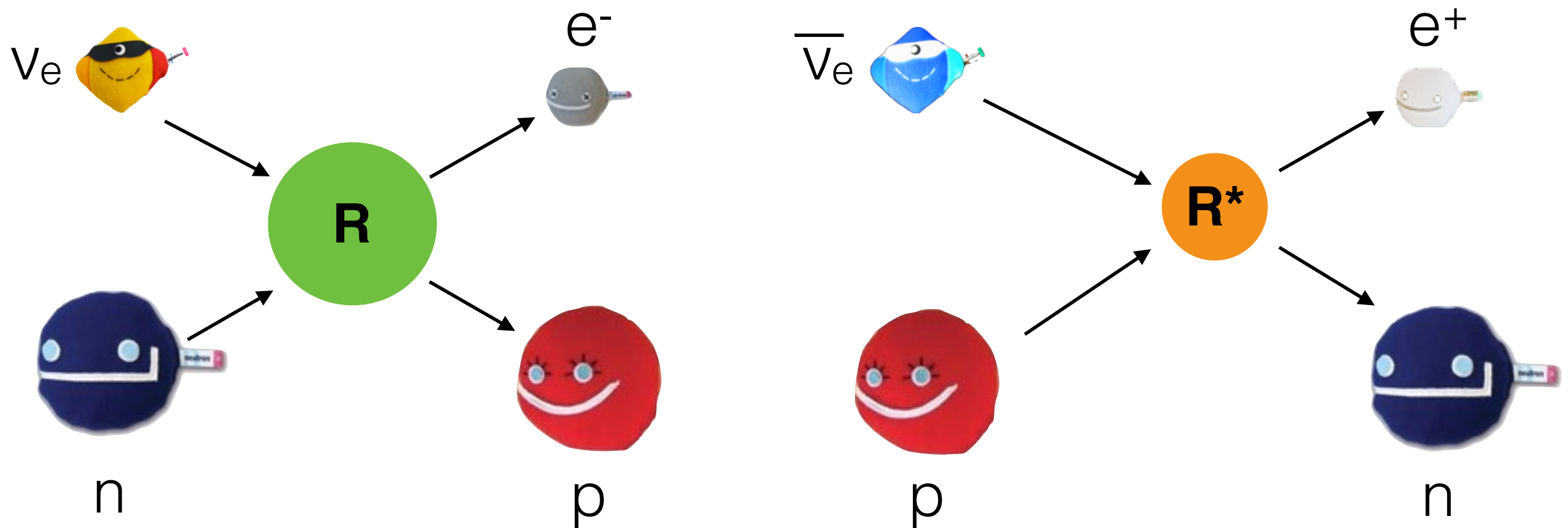
The **sphaleron** process allows part of this **lepton-antilepton asymmetry** to be converted to a **quark-antiquark asymmetry**



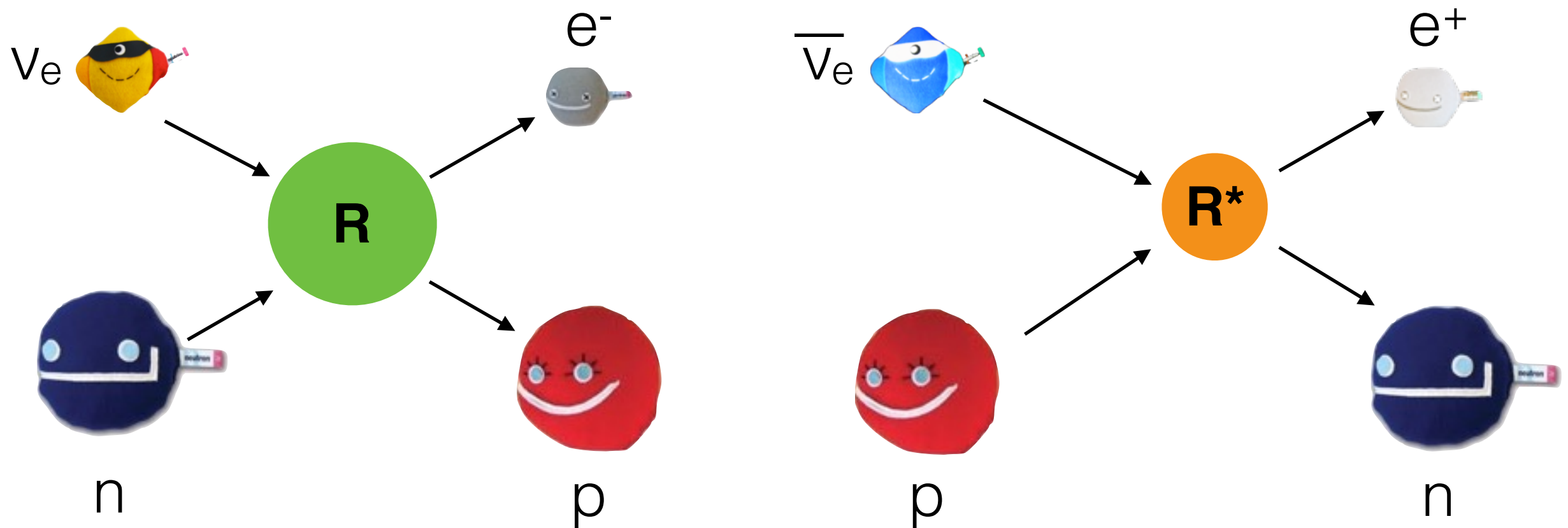
The  $\delta_{\text{CP}}$  term in neutrino mixing measures **CP violation** in neutrino interactions.



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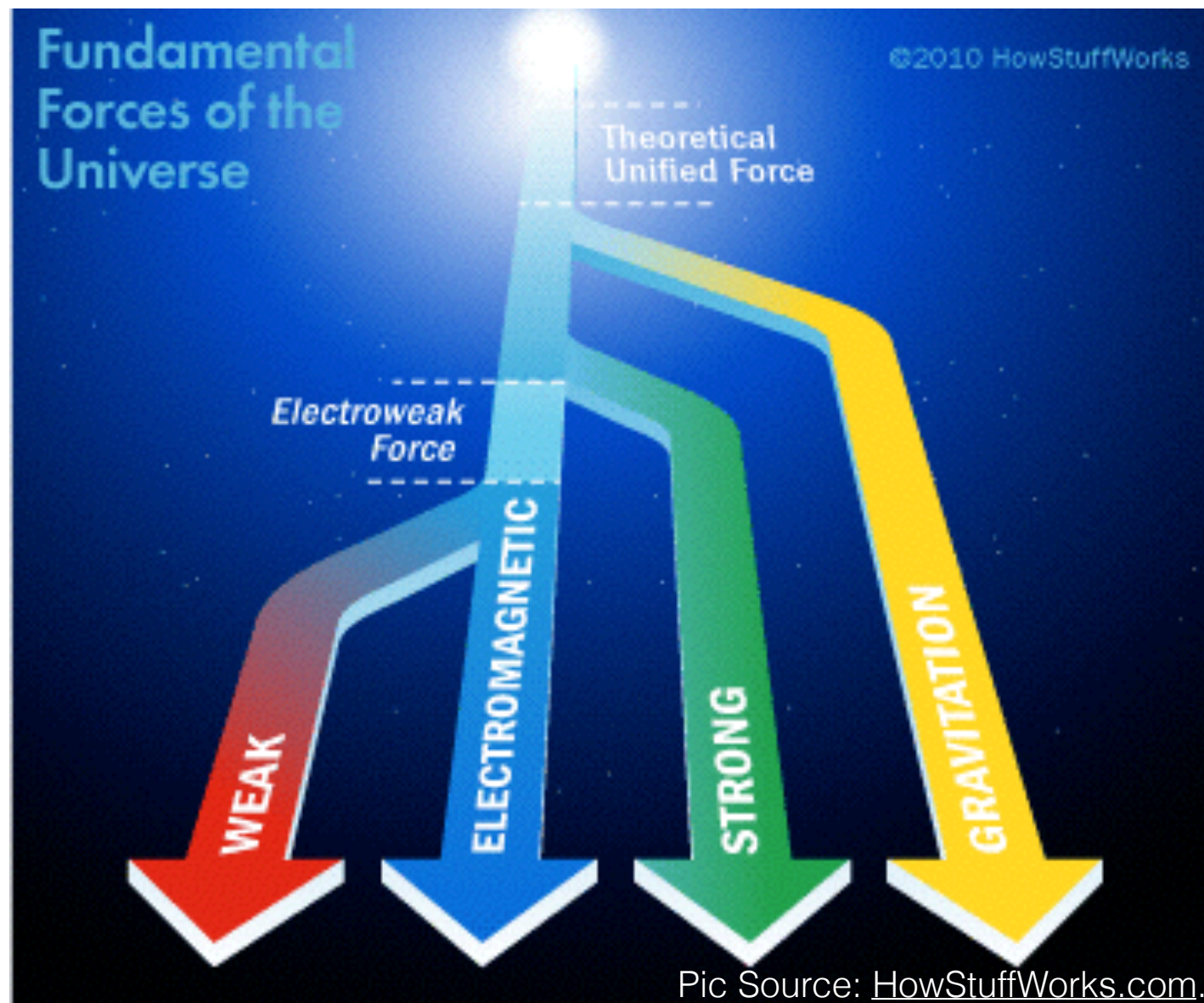
The  $\delta_{\text{CP}}$  term in neutrino mixing measures **CP violation** in neutrino interactions.



$$\text{R} / \text{R}^* = ?$$

We can **compare** these rates to help determine  $\delta_{\text{CP}}$  for light neutrinos

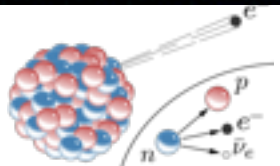
# Theories that seek to explain unification often predict certain neutrino oscillation regimes



Determining **which regimes describe our reality** can narrow down **which theories are valid**

$\delta_{CP}$  **sign( $\Delta m_{32}$ )**

$\theta_{23}$





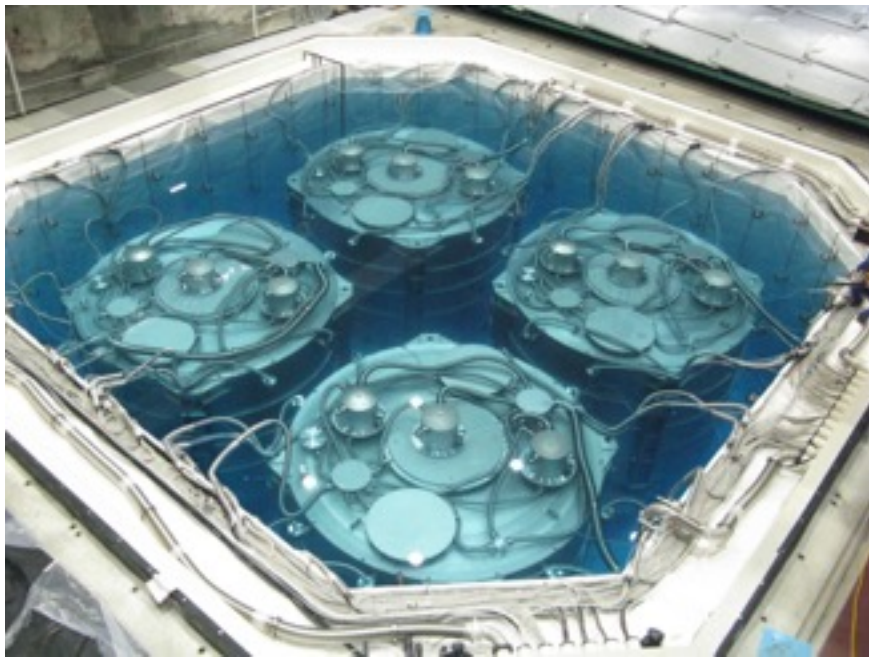
# PART III

## Taking Action



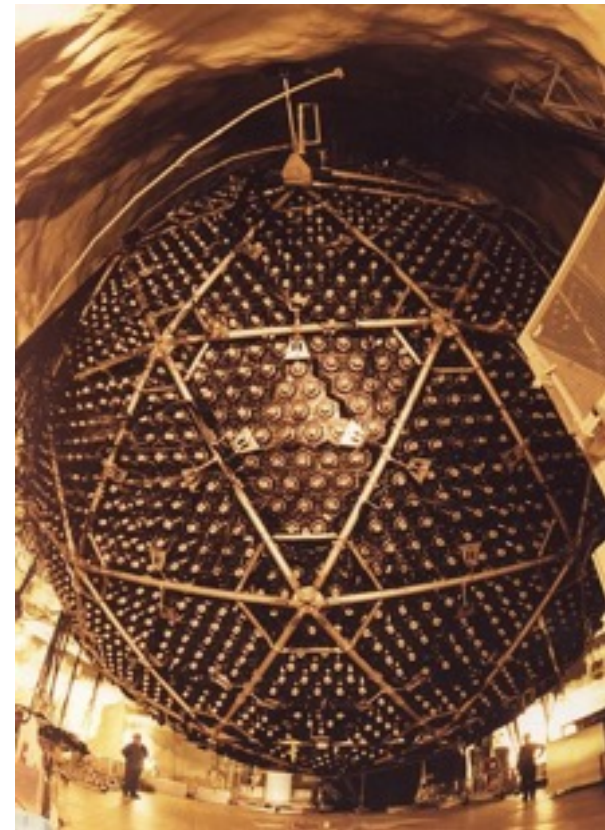
# Solar and reactor neutrino experiments have made groundbreaking progress

Reactor experiments study anti- electron neutrinos produced by nuclear reactors to obtain high precision measurements of  $\theta_{13}$



Days Bay Image Source: [LBL News Center](#)

SNO detector Image Source: [Wikipedia](#)



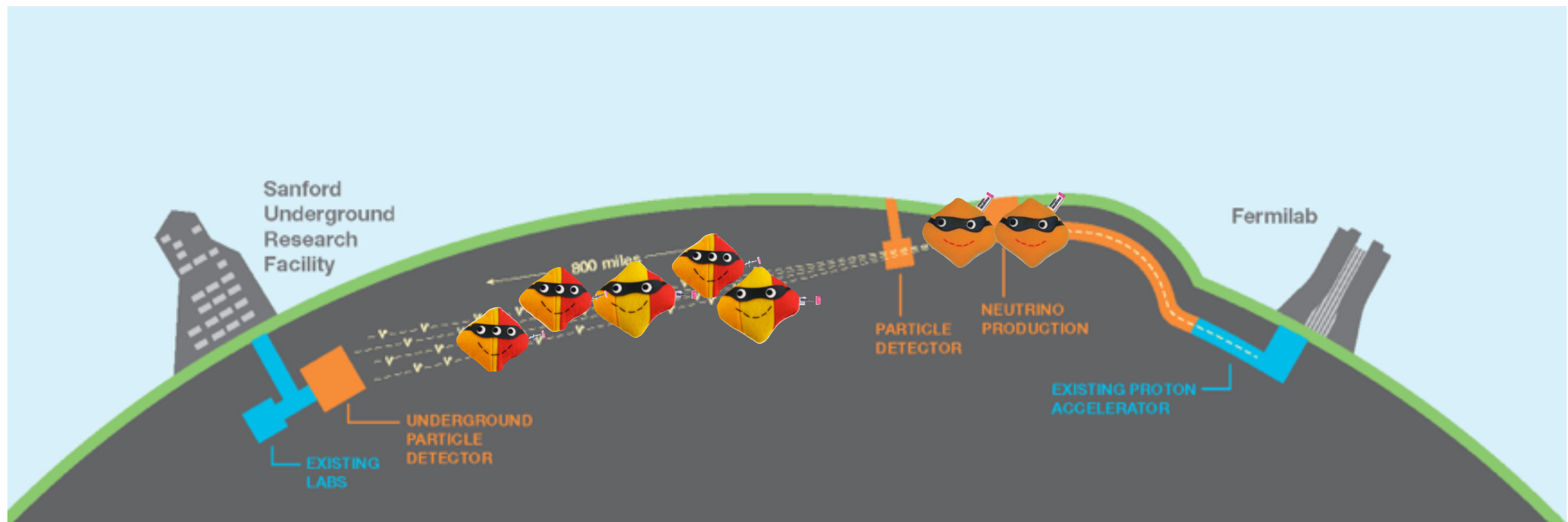
Solar neutrino experiments observe electron neutrinos from the sun to get high-precision values of  $\Delta m_{12}$  and  $\theta_{12}$

These types of experiments have also made measurements of other neutrino oscillation properties!



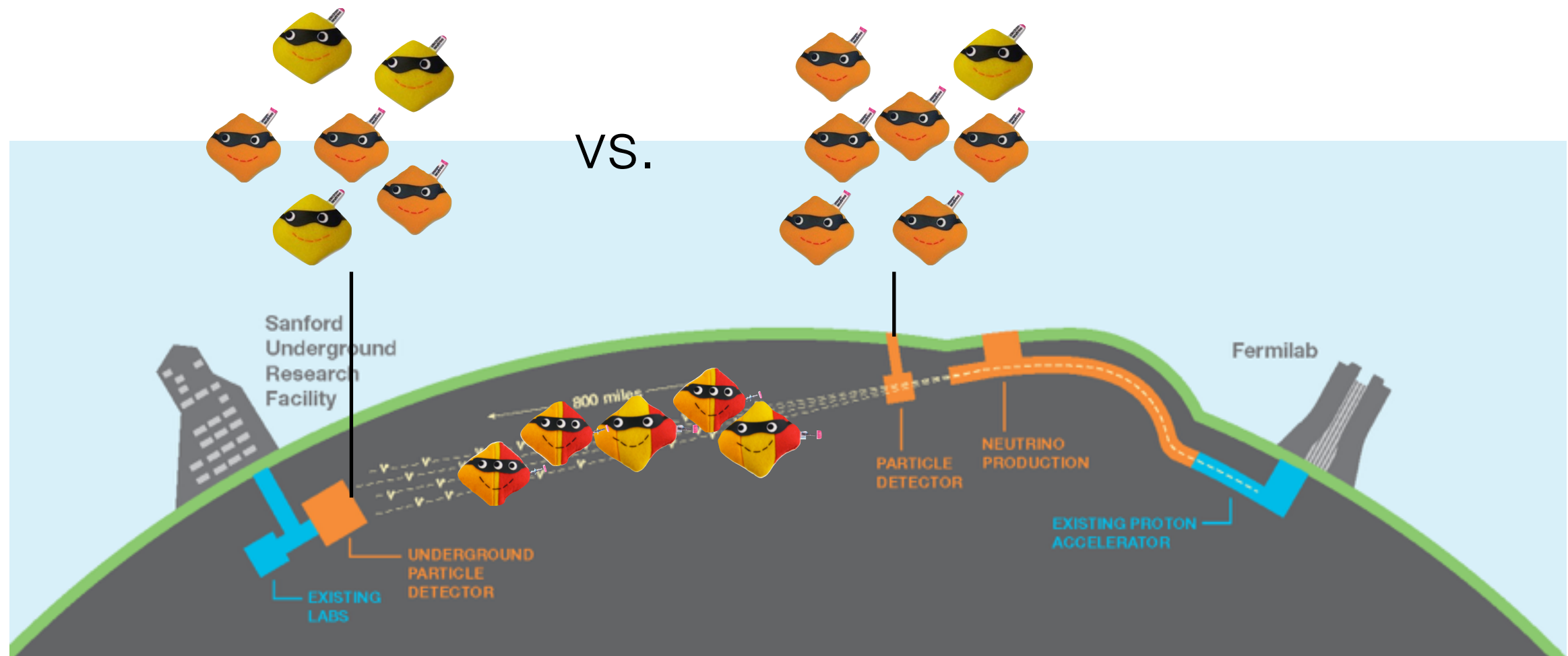
# Long-baseline experiments at accelerators are aiming to finish the fight

In general, a muon neutrino beam is sent through the earth...



In transit, this beam becomes a mix of neutrino flavors...

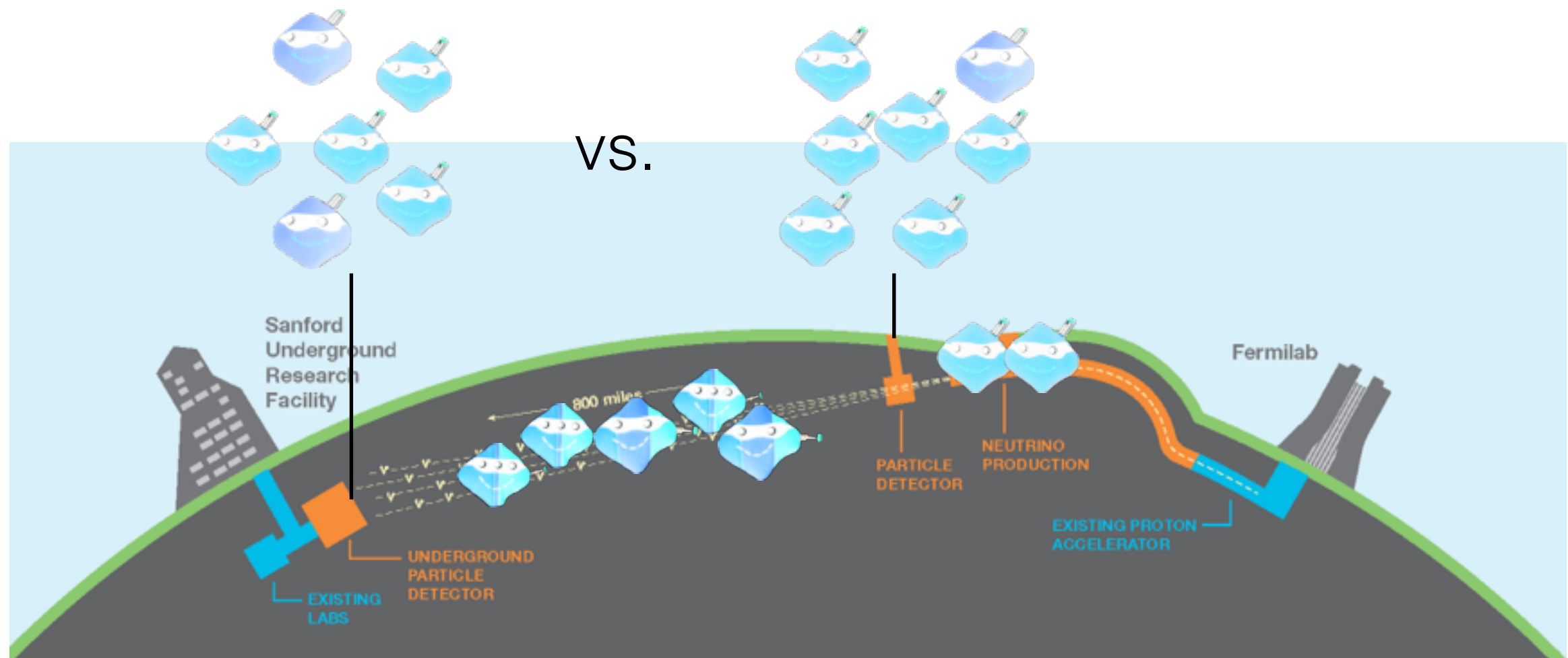
At **near** and **far** detectors, fractions of observed muon and electron neutrino interactions are measured...



And compared to determine likely values of  $\Delta m_{32}$ ,  $\theta_{23}$ , and  $\delta_{CP}$  (among others)

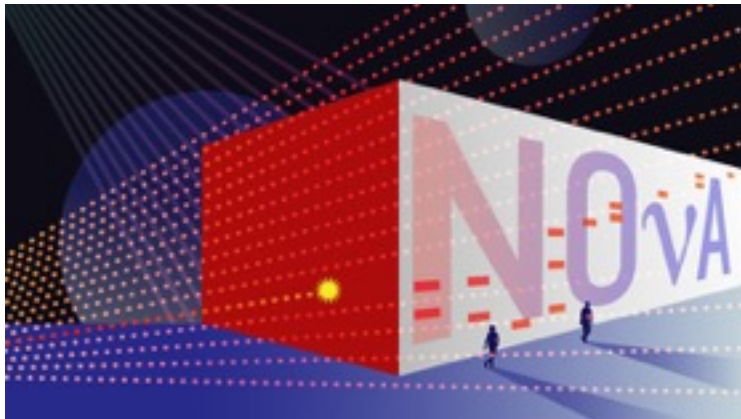


# The same comparisons can be made for beams of antineutrinos

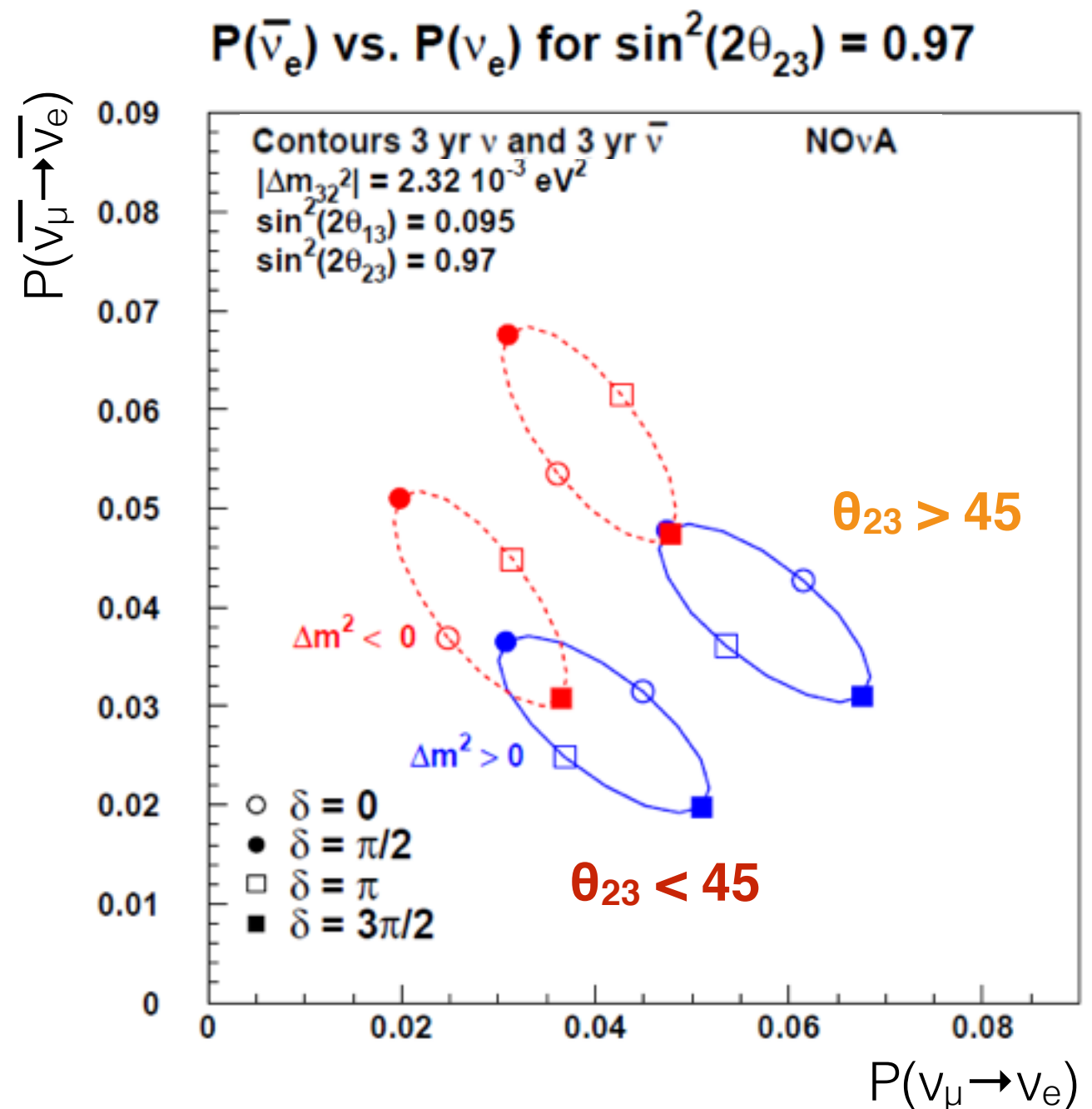


And compared to provide even further restrictions on these  $\Delta m_{32}$ ,  $\theta_{23}$ , and  $\delta_{CP}$  (among others)

# Long-baseline experiments have been designed to take advantage of this



Red and blue curves would be much closer together without the **matter effect**

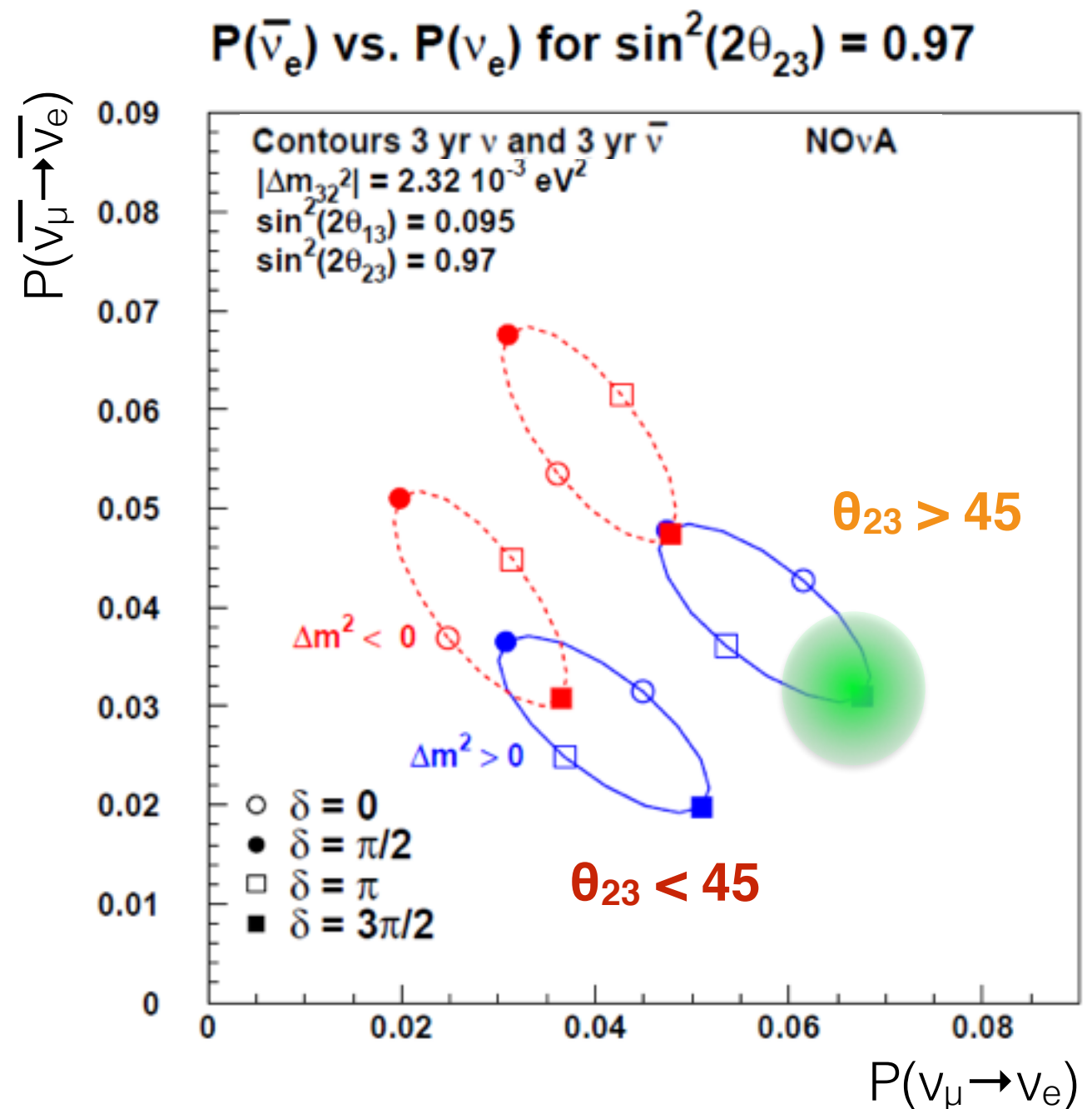


Plot Source: S. Parke, "Determining the Neutrino Mass Hierarchy"

K. Matera

Depending on how kind nature is, a single measurement could resolve several questions at once

For the example point shown, we would be able to determine:



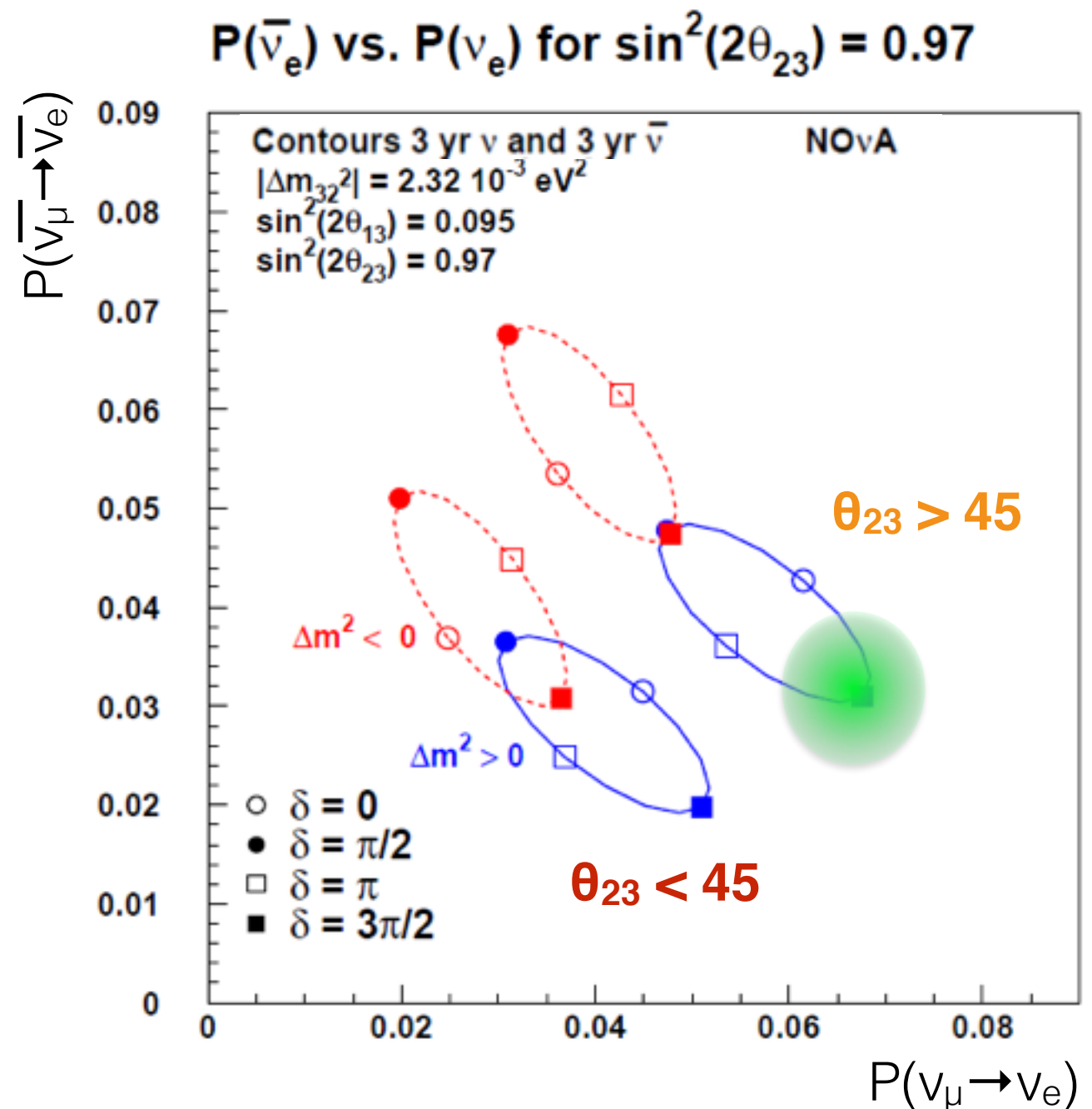
Plot Source: S. Parke, "Determining the Neutrino Mass Hierarchy"

K. Matera

Depending on how kind nature is, a single measurement could resolve several questions at once

For the example point shown, we would be able to determine:

- 1) The mass hierarchy
- 2) That  $\delta_{CP}$  is  $\sim 3\pi/2$
- 3) The  $\theta_{23}$  quadrant





# Combining results of many different experiments will ultimately cut the knot

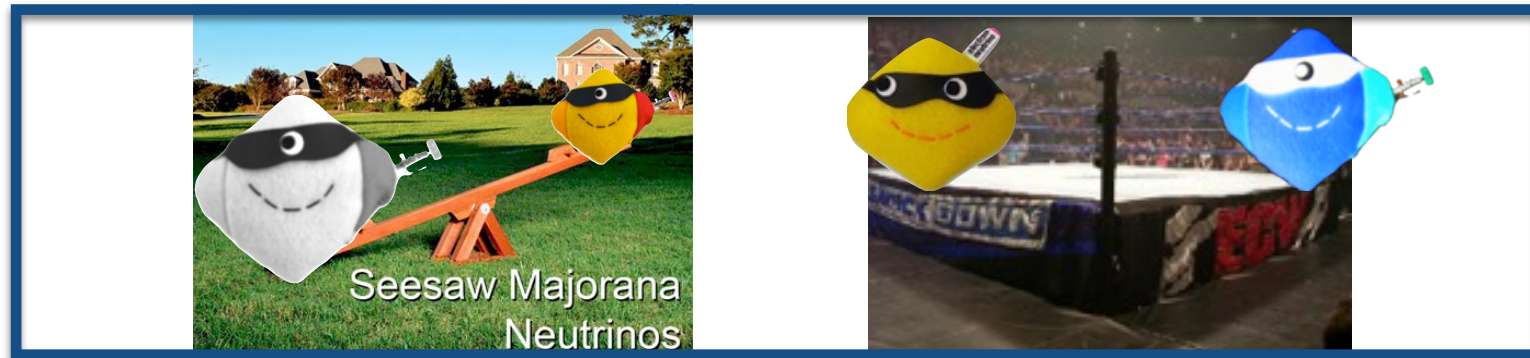


Image (without plushies): [Fermilab Today](#)

Image: [Wikipedia, Wrestling Ring](#)

$\delta_{CP}$



$\theta_{23}$



$\text{sign}(\Delta m_{32})$



Image Source

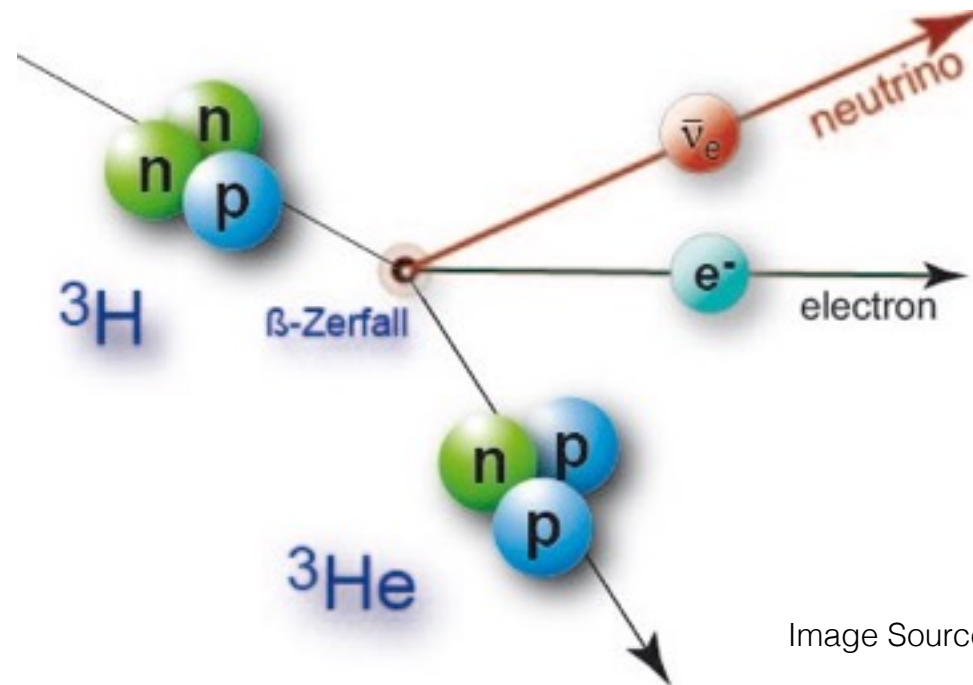
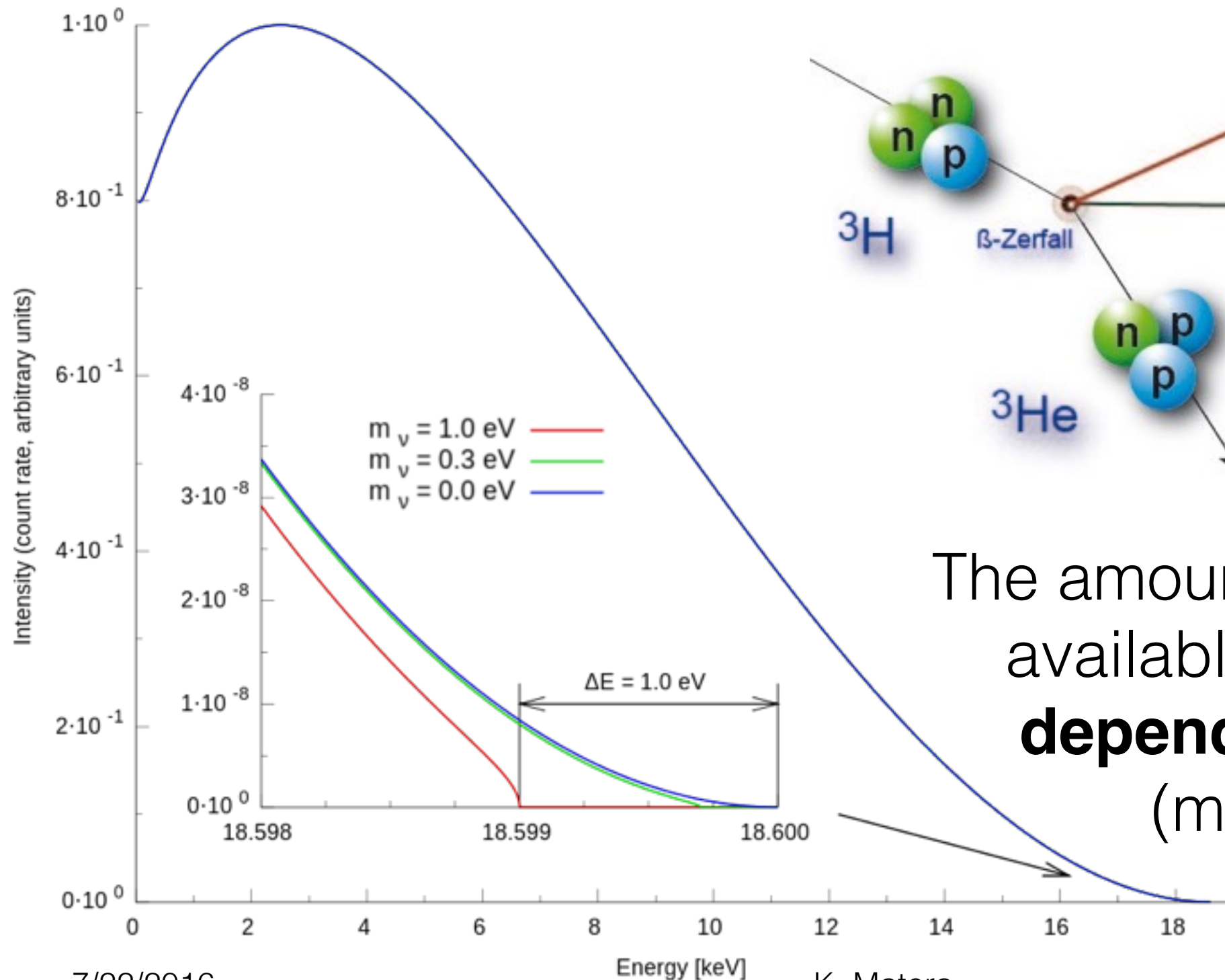
**Thank you!**

K. Matera

7/22/2016

# The End

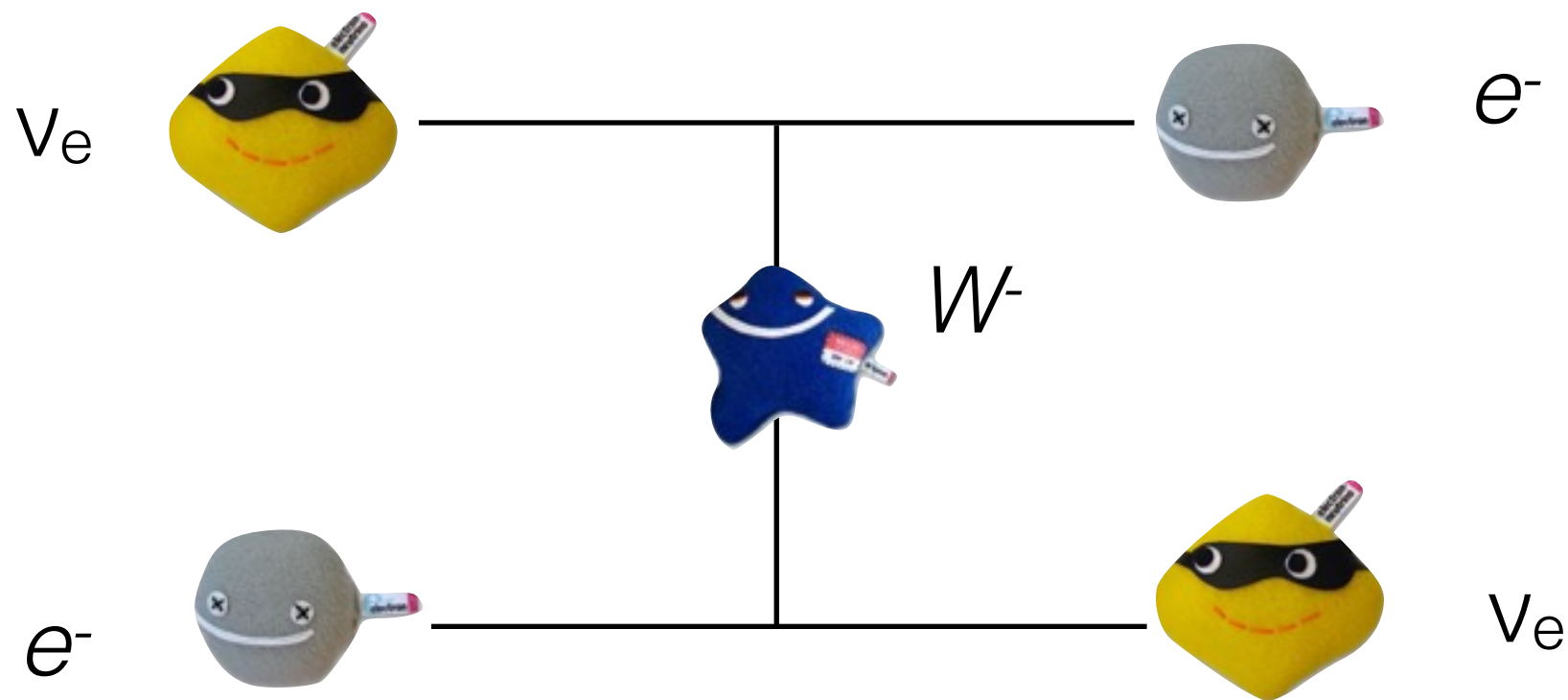
# KATRIN looks at the energy spectrum of tritium beta decays



The amount of kinetic energy available to the neutrino **depends on its mass!**  
(mostly  $m(\nu_1)$ )

# The **matter effect** amplifies the influence of the mass hierarchy

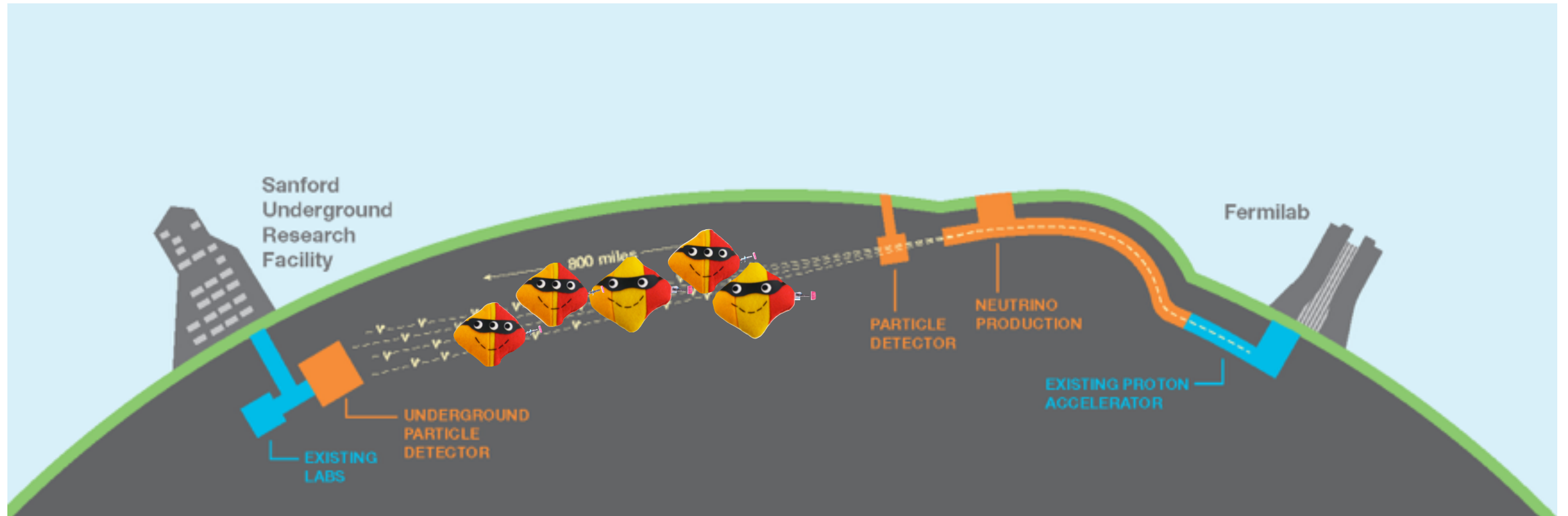
As a beam of neutrinos passes through matter, interactions with electrons “refresh” the  $\nu_e$  component



In fact, at a high enough density, the  $\nu_e$  component would be ‘locked in’, and become an effective mass eigenstate.



In a *normal* (*inverted*) hierarchy:



As a beam of neutrinos passes through **matter**, interactions with electrons **increase** (**decrease**) the **nue fraction**.

At the same time, a beam of anti-neutrinos would see the **anti-nue fraction** **decreased** (**increased**).